Trade Agreements and International Comovements

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Abstract

This paper investigates the effects of signing a trade agreement on the correlations of the business cycle fluctuations of consumption, investment and output between two countries. We construct an international business cycle model with trade costs and we calibrate it to the United States and Mexico in order to estimate the impact of NAFTA on their comovements. Although there exist some discrepancies between the theory and data in the degree of correlation, the direction of change corresponds to the one in the data. Moreover, the financial markets imperfections seem to not have a strong effect on comovements.

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1 Introduction

Globalization and trade agreements are very contemporary subjects and a reality affecting many economic variables. The purpose of this paper is to study the effects of the North American Free Trade Agreement (NAFTA) from the business cycles point of view. More explicitly, we analyze the empirical evidence for the US-Mexico comovements before and after signing NAFTA then we test the ability of the international business cycles model to fit the data.

One could believe that countries with stronger international trade linkages tend to have higher correlated business cycles in consumption but less correlated cycles in investment. The argument behind this statement is that trade integration (or signing a trade agreement) decreases trade costs, facilitating the trade in goods and thus making the consumption paths of the countries involved more correlated. On the other hand a reduction in trade barriers also means that the investment from one country could flow to the other, thus decreasing the correlation. In terms of output the intuition can lead us to two different rational scenarios. Standard trade theory tells us that increased trade leads to more specialization. On one hand, if the specialization induces more inter-industry trade and the shocks driving the business cycles are sector-specific than there is no reason to think that trade intensification would lead to higher comovements in output. On the other hand, if what is intensified is the intra-industry trade, we can expect an increase in the comovement of outputs of the countries involved, induced by the "back-and-forth" trade.

However, this intuition is not supported either by the empirical or theoretical literature. Until now only the relationship between trade barriers and output comovements received direct attention in both empirical and theoretical studies. Frankel and Rose (1998) and Imbs (2004), among others, show that stronger trade linkages are associated with higher correlations of outputs in the data. Kose and Yi (2001, 2006) examine whether these patterns can be explained by the standard international business cycle model. With the calibration they chose (for US, Germany and Japan), the model failed to fit the data.

There is a large branch of literature that studies the economic impact of NAFTA. This literature analyses the impact of the trade agreement on the volume of trade between the countries involved. Krueger (1999) presents empirical evidence that in the case of NAFTA, trade creation was bigger than trade diversion. From a theoretical point of view, this analysis of the implication of NAFTA on the volume of trade is usually done using static applied general equilibrium models based on increasing returns and imperfect competition (see Kehoe (2003) for a survey). However, the static nature of these models makes them unfit for our purpose. To understand the effects of NAFTA on the countries' business cycles, one needs to analyze the behavior of macroeconomic aggregates. As Kehoe and Kehoe (1994) emphasized in their paper, accounting for changes in productivities is vital for capturing changes in the macroeconomic aggregates.

We therefore use a business cycle model with productivity shocks to analyze the comovements in the main economic aggregates: consumption, investment and output. Our analysis is thus related to the international business cycle literature. The focus of this literature is on two major discrepancies between the data and the predictions of the international business cycle models. The international business cycle models with complete markets predict negative cross-country correlations of investment (while in the data these correlations are positive) and cross-country correlations for consumption much higher than for output (while in the data the opposite is true).¹

Many economists tried to solve these so called "puzzles", by introducing various frictions in the international business cycle models. Among the frictions considered, the most frequent are the transportation costs² and incomplete financial markets.³ The expected consequences of these frictions are a reduction in the correlations of consumption and an increase in the correlations of output and investment. Several papers show that these frictions are able to reduce the existing anomalies, and even completely eliminate them.⁴ Nevertheless these papers focus on the US-Canada or US-Europe aggregate data and therefore their models are calibrated for these particular situations.

The focus of the present paper is on the US and Mexico and the effects of NAFTA on these countries' consumption, investment and output comovements. A stylized, simplified way of incorporating trade agreements into theoretical economic models is in the form of changes in trade costs between countries.⁵ Therefore we introduce a trade cost in intermediate goods in the standard international business cycle model (a la Backus, Kehoe, Kydland (1995)) and simulate the signing of NAFTA by a reduction in this cost.

We first look at the case of complete financial markets and show that a decrease in the trade cost is able to reproduce the direction of changes in comovements but not the size. Second, we investigate the effects of financial markets frictions by analyzing two particular incomplete markets environments: one in which the only traded asset is an one-period bond, the other in which there is financial autarky. Our results show that taking into account the financial markets frictions improves the results, reducing the quantitative gap between the model and the data.

The paper is organized as follows: we start by presenting the empirical evidence in section 2, then set up the model in section 3. Section 4 describes the calibration of the model and section 5 presents the results. Finally, the last section concludes and presents lines for future research.

2 Empirical evidence

The objective of this section is to analyze the changes in the correlations of consumption, investment and output for the case of USA and Mexico. Our main focus is on the patterns of the correlations around the year 1994 when the North American Free Trade Agreement (NAFTA) came into force.

Although NAFTA was signed by USA, Mexico and Canada, we present here only the empirical evidence for the case of USA and Mexico while the evidence regarding the other two pairs of countries (USA-Canada, Canada-Mexico) is presented in Appendix 2. We made this choice for two reasons. First since USA and Canada have had intense trade-relationship even before 1994 (for example, the Canadian Free Trade Agreement signed between Canada and USA came

¹See Backus, Kehoe, Kydland (1992) for details.

²See Mazzenga and Ravn (2002)

³See Kose and Yi (2001, 2006), Kehoe and Perri (2002), Heathcote and Perri (2002)

⁴See Kehoe and Perri (2002).

 $^{{}^{5}}$ Since a trade agreement represents more than a reduction in the trade barriers, this paper represents the first step in our intent to see all the effects that trade liberalization can have over the international comovements.

into force in 1989) one would not expect any strong impact of NAFTA on their macroeconomic paths. Second, the case of Mexico-Canada is not so interesting because the volume of trade between Canada and Mexico is only 4 percent of existing trade between US and Mexico.⁶

Table 1 displays the evolution of the cross-country correlations of consumption, investment and output. To compare correlations before and after signing NAFTA, we compute correlations over equal periods of time, before and after $1994.^7$

[Table 1.]

Comparing the correlations for USA and Mexico before and after 1994 we observe a clear increasing tendency in all the three macroeconomic variables considered. However, this evidence is not enough to conclude that there is any possible impact of NAFTA. The table above only shows that the correlations after 1994 are higher than before. This might be just the natural consequence of a constant increase in the correlations. To eliminate this possibility we analyze the evolution of the comovements in time by considering correlations over a "flying" 5-year window. *Figure 1* illustrates the correlations in time for consumption, investment and output. The smoothed lines represent the trends of the comovements while the more volatile line, the fluctuations around this trend.⁸ The figure shows, for all the variables, a clear change in the tendency in 1992, the year when the representatives of Canada, Mexico and the USA concluded their negotiations for signing the North American Free Trade Agreement⁹.

[Figure 1]

We interpret these results as an evidence of NAFTA's impact on the comovements between USA and Mexico. The rest of the paper investigates the ability of the standard international business cycle model to generate these results. The following section describes in detail the model we use under all the financial markets structures.

3 The International Business Cycle Model

We consider a standard neoclassical growth model "a la" Backus, Kehoe, Kydland (1995) (BKK henceforth) under three different financial markets structures: complete markets, bond economy and financial autarky. The economy consists of two countries that produce different, imperfectly substitutable intermediate goods which can be traded between countries. After the trade, each country uses as final good a combination of the domestic and imported goods. The difference from the BKK model consists in the "iceberg" trade cost τ we introduce in this model. The rough way in which we simulate a trade agreement in this economy is through reducing τ . Two new features are studied here. First, we investigate how the correlations in consumption,

⁶The static applied general equilibrium models also conclude that NAFTA is likely to have a strong positive impact on the Mexican economy, only a small impact on the US economy and almost no impact on the Canadian economy. See Kehoe and Kehoe (1994) for more details.

Appendix 3 presents data on trade and comparative statistics.

⁷See Appendix 1 for a more detailed description of data.

⁸We applied Hodrick-Prescott filter to abtain the trend and the fluctuations.

⁹Figure 2 and Figure 3 in the Appendix 2 present the evolution of the correlations between Mexico and Canada, USA and Canada respectively.

investment and output evolved in time after the reduction in trade cost. Second, due to the specifics of the case we want to study, USA-Mexico, Backus et all.(1995) or Mazzenga and Ravn (2002) 's assumption that the equilibrium is symmetric cannot be applied. This complicates the way the model is solved and the calibration we make in order to simulate the results. In the following subsections we will present in more detail the model.

3.1 Common setup

Preferences

Each country, i, is represented by a continuum of identical, infinitely lived agents having the preferences characterized by the following expected utility function

$$u_i = E_0 \sum_{t=0}^{\infty} \beta_i^t U_i(c_{it}, 1 - n_{it}) \quad , i = 1, 2,$$
(1)

where c_{it} and n_{it} represents per capita consumption and time devoted to labor in country i and β is the intertemporal discount factor. The utility function is $U_i(c_i, 1 - n_i) = [c_i^{\mu_i}(1 - n_i)^{1-\mu_i}]^{1-\gamma_i}/(1-\gamma_i)$, where μ_i represents the consumption share and γ_i the coefficient of relative risk aversion. The endowment of time is normalized to 1 in each period t. Therefore $(1 - n_{it})$ represents time devoted to leisure, per capita, in country i.

We denote by Ψ the relative mass of population of country 1.

Technologies

Intermediate goods

Each country specializes in the production of a single intermediate good, labeled a for country 1 and b for country 2. Production of these goods takes place in each country using as inputs domestic capital, k, and domestic labor, n (both internationally immobile) and it is affected by the technology shocks, z. Some of the intermediate goods are used domestically for the production of final goods while the rest is exported. This gives rise to the resource constraints:

$$a_{1t} + a_{2t} = y_{1t} = z_{1t}F(k_{1t}, n_{1t})$$

$$\tag{2}$$

$$b_{1t} + b_{2t} = y_{2t} = z_{2t}F(k_{2t}, n_{2t}) \tag{3}$$

in country 1 and 2 respectively, where $F(k_i, n_i) = k_i^{\theta_i} n_i^{1-\theta_i}$ and z_{it} represents the productivity shock specific to country *i*. The productivity shocks follow the process $z_{it+1} = A_i z_{it} + \varepsilon_{it+1}$, where $corr(A_i, A_j) \neq 0$ and the innovations ε_{1t+1} and ε_{2t+1} are correlated. The variable y_{it} represents output per capita in country *i* and a_{2t}, b_{1t} represent the quantities exported to country 2 (good *a*), respectively to country 1 (good *b*), also per capita.

Final goods

Consumption and investment in each country are composites of the foreign and domestic intermediate goods:

$$c_{1t} + x_{1t} = G(a_{1t}, b'_{1t}) \tag{4}$$

$$c_{2t} + x_{2t} = G(b_{2t}, a'_{2t}) \tag{5}$$

where G(a, b) is the Armington aggregator: $G(a, b) = [\omega_1 a^{1-\alpha} + \omega_2 b^{1-\alpha}]^{\frac{1}{1-\alpha}10}$ and b'_{1t}, a'_{2t} represents the quantities imported by country 1 and 2 respectively. Parameters $\alpha, \omega_1, \omega_2$ are positive, $\sigma = \frac{1}{\alpha}$ represents the elasticity of substitution between goods a and b and ω_1, ω_2 represent the home, and, respectively, foreign bias in the composition of domestically produced final goods.

The law of motion for the capital stock is given by

$$k_{jt+1} = (1 - \delta_j)k_{jt} + x_{jt} \tag{6}$$

where $\delta_j \in (0, 1)$ is the depreciation rate and x_{jt} represents the amount of final good devoted to investment in country j.

Trade cost

As we said at the beginning, we introduce a trade cost, $\tau \in [0, 1]$ in this economy. This cost affects international trade in intermediate goods in the following way: if a quantity q is exported, only a fraction $(1 - \tau)$ of q reaches the destination. Therefore the feasibility conditions for the final goods could be rewritten as:

$$c_{1t} + x_{1t} = G(a_{1t}, (1 - \tau)b_{1t}) \tag{7}$$

$$c_{2t} + x_{2t} = G(b_{2t}, (1 - \tau)a_{2t}) \tag{8}$$

3.2 Equilibrium

In this section we describe the equilibrium allocations under all the financial market structures we consider. We start with the complete markets, then we describe the equilibrium under bond economy and the last, the financial autarky specification.

3.2.1 Complete markets

Defining a competitive equilibrium for this economy with complete contingent claims markets is straightforward but notationally burdensome. We prefer to determine the equilibrium allocations by exploiting their welfare properties. Each country is populated by identical consumers who will choose identical consumption and investment plans in the equilibrium. In the equilibrium consumers use contingent claims to diversify country-specific risk across states of nature. By doing so, consumers end up equating the marginal utility of consumption across countries for each state of nature thus, each equilibrium allocation is Pareto optimal. Therefore the equilibrium in this model is that Pareto optimal allocation in which the consumers from the same country choose the same allocations. For this reason we can consider a representative consumer for each one of the economies. Moreover, if we consider the continuum of agents in country 1 having the relative mass Ψ (relative to the total of consumers in the economy), the relative mass of country 2 will be $1 - \Psi$.¹¹ The equilibrium allocation under complete markets specification is thus fully characterized in the following proposition.

¹⁰This is like having the preferences defined over goods a and b. But for computational simplicity we call goods a and b "intermediate goods" and the aggregation of both (i.e. G(a, b)) we call it the final good.

¹¹Taking into account that we want to calibrate this model for the case of USA and Mexico, we cannot impose the symmetry. Thus $\Psi \neq 0.5$

Proposition 1: The allocation $(a_{1t}^*, a_{2t}^*, b_{1t}^*, b_{2t}^*, c_{1t}^*, c_{2t}^*, x_{1t}^*, x_{2t}^*, k_{1t}^*, k_{2t}^*, n_{1t}^*, n_{2t}^*)$ is an equilibrium allocation of the economy, given the transportation cost τ , if it solves the following problem:

$$\max \Psi u(c_{1t}, 1 - n_{1t}) + (1 - \Psi)u(c_{2t}, 1 - n_{2t}) \text{ subject to}$$
(9)

$$c_{1t} + x_{1t} = G(a_{1t}, (1 - \tau)b_{1t})$$

$$c_{2t} + x_{2t} = G(b_{2t}, (1 - \tau)a_{2t})$$

$$\Psi a_{1t} + (1 - \Psi)a_{2t} = \Psi z_{1t}F(k_{1t}, n_{1t})$$

$$\Psi b_{1t} + (1 - \Psi)b_{2t} = (1 - \Psi)z_{2t}F(k_{2t}, n_{2t})$$

$$k_{1t+1} = (1 - \delta_1)k_{1t} + x_{1t}$$

$$k_{2t+1} = (1 - \delta_2)k_{2t} + x_{2t}$$

3.2.2 Bond economy

We consider now an economy where a single non-contingent bond is traded at every period t. The bond traded at t matures at t + 1, when it pays one unit of the intermediate good a. Let B_{it}^* denote the quantity of bonds traded, in equilibrium, at time t in country i and r_t be its price measured in units of the intermediate good a^{12} . Let also w_i, v_i denote the prices of labor and capital, respectively, in country i, expressed in terms of the intermediate good produced by country i. The intermediate goods prices are q_{1t}^a, q_{1t}^b in the first country, and q_{2t}^a, q_{2t}^b in the second country. They are expressed in units of the consumption good produced in the respective country. Using these prices and the rest of notation from the previous paper we can write the definition for a competitive equilibrium¹³:

Definition 1: The allocation

 $(a_{1t}^*, a_{2t}^*, b_{1t}^*, b_{2t}^*, c_{1t}^*, c_{2t}^*, x_{1t}^*, x_{2t}^*, k_{1t}^*, k_{2t}^*, n_{1t}^*, n_{2t}^*, B_{1t}^*, B_{2t}^*, q_{1t}^a, q_{2t}^a, q_{1t}^b, q_{2t}^b, w_{1t}, w_{2t}, v_{1t}, v_{2t}, r_{1t}, r_{2t})$

is a **competitive equilibrium** in this economy if and only if:

(1) $(c_{1t}^*, x_{1t}^*, n_{1t}^*, B_{1t+1}^*)_{t\geq 0}$ maximizes the intertemporal utility function

$$\sum_{t=0}^{\infty} \beta_1^t u(c_{1t}, 1 - n_{1t})$$

 $^{^{12}}$ The "no arbitrage" condition requires that the price of the bond (expressed in units of good a) must be the same in both countries.

¹³Due to the market imperfections presented in this model the welfare theorems no longer hold. In order to determine the steady state and the dynamics of this economy we need to solve the competitive equilibrium problem.

subject to the budget constraints

$$c_{1t} + x_{1t} + r_t q_{1t}^a B_{1t+1} = q_{1t}^a (w_{1t} n_{1t} + v_{1t} k_{1t} + B_{1t}), \quad \forall t = 1, \infty$$

$$k_{1t+1} = (1 - \delta_1) k_{1t} + x_{1t}, \quad \forall t = 1, \infty$$

$$k_{10}, B_{10} \text{ - given}$$

$$(10)$$

(2) $(c_{2t}^*, x_{2t}^*, n_{2t}^*, B_{2t+1}^*)_{t\geq 0}$ maximizes the intertemporal utility function

$$\sum_{t=0}^{\infty} \beta_2^t u(c_{2t}, 1 - n_{2t})$$

subject to the budget constraints

$$c_{2t} + x_{2t} + r_t q_{2t}^a B_{2t+1} = q_{2t}^b (w_{2t} n_{2t} + v_{2t} k_{2t}) + q_{2t}^a B_{2t}, \quad \forall t = 1, \infty$$

$$k_{2t+1} = (1 - \delta_2) k_{2t} + x_{2t}, \quad \forall t = 1, \infty$$

$$k_{20}, B_{20} - \text{given}$$

$$(11)$$

(3) $(k_{1t}^*, n_{1t}^*, a_{1t}^*, a_{2t}^*)_{t \ge 0}$ solves country 1's intermediate-good-firm's problem:

$$\max\left[z_{1t}(k_{1t})^{\theta}(n_{1t})^{1-\theta} - (w_{1t}n_{1t} + v_{1t}k_{1t})\right]$$
(12)

(4) $(k_{2t}^*, n_{2t}^*, b_{1t}^*, b_{2t}^*)_{t\geq 0}$ solves country 2's intermediate firm's problem:

$$\max[z_{2t}(k_{2t})^{\theta}(n_{2t})^{1-\theta} - (w_{2t}n_{2t} + v_{2t}k_{2t})]$$
(13)

(5) $(a_{1t}^*, b_{1t}^*)_{t\geq 0}$ solves country 1's final-good-firm's problem:

$$\max G(a_{1t}, (1-\tau)b_{1t}) - q_{1t}^a a_{1t} - q_{1t}^b b_{1t}$$
(14)

(6) $(a_{2t}^*, b_{2t}^*)_{t\geq 0}$ solves country 2's final-good-firm's problem:

$$\max G(b_{2t}, (1-\tau)a_{2t}) - q_{2t}^a a_{2t} - q_{2t}^b b_{2t}$$
(15)

(7) all markets clear :

(a) market for intermediate good produced in country 1:

$$\Psi a_{1t} + (1 - \Psi)a_{2t} = \Psi z_{1t}F(k_{1t}, n_{1t}) \tag{16}$$

(b) market for intermediate good produced in country 2:

$$\Psi b_{1t} + (1 - \Psi)b_{2t} = (1 - \Psi)z_{2t}F(k_{2t}, n_{2t})$$
(17)

(c) market for final good from country 1:

$$c_{1t} + x_{1t} = G(a_{1t}^*, (1 - \tau)b_{1t}^*)$$
(18)

(d) market for final good from country 2:

$$c_{2t} + x_{2t} = G(b_{2t}^*, (1 - \tau)a_{2t}^*) \tag{19}$$

(e) bond market:

$$\Psi B_{1t} + (1 - \Psi) B_{2t} = 0 \tag{20}$$

where $G(a,b) = [\omega_1 a^{1-\alpha} + \omega_2 b^{1-\alpha}]^{\frac{1}{1-\alpha}}$, and Ψ represents the relative weight of country 1 in the system.

(8) initial conditions:

$$k_{i0}^* = k_{i0}, \quad i = 1, 2$$

 $B_{i0}^* = B_{i0}, \quad i = 1, 2$

3.2.3 Financial autarky

In the economy with no trade in financial assets, the definition of the equilibrium changes from the bond economy as described below:

A. the budget constraints for utility maximization problems change as follows:

(i) (10) is replaced by

$$c_{1t} + x_{1t} = q_{1t}^{a}(w_{1t}n_{1t} + v_{1t}k_{1t}), \quad \forall t = 1, \infty$$
$$k_{1t+1} = (1 - \delta_1)k_{1t} + x_{1t}, \quad \forall t = 1, \infty$$
$$k_{10} - \text{given}$$

1. (ii) (11) is replaced by

$$c_{2t} + x_{2t} = q_{2t}^{b}(w_{2t}n_{2t} + v_{2t}k_{2t}), \quad \forall t = 1, \infty$$
$$k_{2t+1} = (1 - \delta_2)k_{2t} + x_{2t}, \quad \forall t = 1, \infty$$
$$k_{20} - \text{given}$$

B. the market clearing condition for bonds disappears (equation (20))

C. the initial condition for bonds disappear.

All the rest remains the same (intermediate-good-firm's problems, final-good-firm's problems and the rest of the market clearing conditions).

In order to figure out the decision rules in equilibrium, in all the financial markets specifications, we solve for the deterministic steady state of the model and approximate the dynamics of the model in response to exogenous productivity shocks. We do so by linearizing the first order conditions around the steady state, as described in King, Plosser, Rebelo (1988).

4 Calibration

The complete list of parameters we have to calibrate is the following: the intertemporal discount factor β_i , the consumption share in the utility function μ_i , the degree of risk aversion γ_i , the technology coefficient θ_i , the depreciation rate δ_i , the home and foreign bias in final good production ω_{i1} and ω_{i2} , the elasticity of substitution between intermediate goods α_i , the persistence matrix for technology shocks A and the variance-covariance matrix of shocks V.

Some of these parameters can be estimated from the available data, others we borrow from other papers. We will start with the last ones: the value for the relative risk aversion coefficient is taken from BKK: $\gamma_1 = \gamma_2 = \gamma = 2$. In the next section we run a sensitivity analysis to see if this value match the best our predictions with the empirical evidence we have. Also we take the discount factor to be 0.99 in both countries (the value normally used in the literature. See BKK(1995), Kehoe, Perri(2002))

To estimate the rest of the parameters in the model we use data from OECD (Quarterly National Accounts) for the period 1980:1-2002:4. First we estimate the time devoted to work (n) in steady state: using the first moment approximation we obtain a share of 0.34 in USA and 0.36 in Mexico. Also from Kaldor' stylized facts we know that $\frac{k}{y}$ is roughly constant. Our estimation is: $(\frac{k}{y})_{USA} = 13.28, (\frac{k}{y})_{MEX} = 11.13$. The estimates for consumption shares of output are $(\frac{c}{y})_{USA} = 0.66, (\frac{c}{y})_{MEX} = 0.68.^{14}$

To approximate the labor share we use the following formula (as in Gollin (2002)):

$$labor_share = \frac{\frac{employees_compensation}{nr_employees} \cdot work_force}{GDP}$$

With this approximation we obtain an average labor share of 0.64. Therefore

$$\theta_1 = \theta_2 = \theta = 0.36.$$

Having these estimates and the first order conditions from above we can determine the depreciation rates:

$$\delta = \frac{1 - \frac{c}{y}}{\frac{k}{y}} \Rightarrow \delta_{USA} = 0.025; \ \delta_{MEX} = 0.028.$$

Also using first order conditions we can estimate the consumption share in the household's utility, μ :

$$\mu = \frac{1}{1 + \frac{1-\theta}{\theta} \frac{1-n}{c} \frac{k}{n} (\frac{1}{\beta} - 1 + \delta)} \Rightarrow \mu_{USA} = 0.34; \ \mu_{MEX} = 0.38;$$

¹⁴For the US we use the estimated ratios capital-to-output, consumption-to-output from Neumeyer and Perri (2005). For Mexico, we estimate the ratios using capital, consumption and GDP series from Heathcote and Perri (2004).

We calibrate the relative mass of the countries (Ψ) such that the commercial deficit USA-Mexico computed from the model to fit the data. In this way, $\Psi = \Psi_{USA} = 0.61, 1 - \Psi = \Psi_{MEX} = 0.39$.

Table 2 summarizes the values of the parameters used in our experiments.

[Table 2.]

We also consider the steady state share of imports to outputs as being constant, and we estimate it from the available data, before and after signing NAFTA: $s_{USA}(before) = 0.10$, $s_{MEX}(before) = 0.15$, $s_{USA}(after) = 0.13$, $s_{MEX}(after) = 0.29$. Using these ratios and the above values we can calibrate the foreign bias from Armington aggregator and the elasticity of substitution between intermediate goods. We obtain them as functions of τ :¹⁵

elasticity of substitution from Armington aggregator α

$$\begin{cases} 1 - q_1^a (\frac{1-s_1}{q_1^a})^{\alpha_1} = q_1^b (\frac{s_1}{q_1^a})^{\alpha_1} (1-\tau)^{\alpha_1-1} \\ 1 - q_2^b (\frac{1-s_2}{q_2^b})^{\alpha_2} = q_2^a (\frac{s_2}{q_2^b})^{\alpha_2} (1-\tau)^{\alpha_2-1} \end{cases}$$

weights from Armington aggregator ω_{ij}^{16}

$$\begin{cases} \omega_{11} = \frac{[(\frac{1}{s_1} - 1)*(\frac{q_1^{\alpha}(1-\tau)}{q_1^b})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}}{1 + [(\frac{1}{s_1} - 1)*(\frac{q_1^{\alpha}(1-\tau)}{q_1^b})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}} \\ \omega_{21} = \frac{[(\frac{1}{s_2} - 1)*(\frac{q_2^{b}(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}}{1 + [(\frac{1}{s_2} - 1)*(\frac{q_2^{b}(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}} \end{cases}$$

It is interesting to observe here how the elasticity of substitution and the weights each country puts on its domestically produced good change with the intensity of the trade barriers (τ). Higher trade barriers are associated with lower import shares, higher elasticities of substitution and lower weights the imports receive in the final good production. We can interpret this as a link between trade intensity and specialization: lower trade barriers generate more intensive trade and in the same time induce a decrease in the elasticity of substitution. The intermediate goods of the two countries become "more" complements and this makes each country specialize in one good.¹⁷

The last parameters we need to estimate are related to the productivity shocks processes. First we compute the total factor productivity in each one of the countries using Solow residuals:

$$\ln TFP = \ln(y) - capital_share \cdot \ln(k) - labor_share \cdot \ln(L),$$

¹⁵For a more detailed example of calibation for the Armington aggregator coefficients see Appendix 4. ¹⁶ $G_i(a,b) = [\omega_{i1}a^{1-\alpha} + \omega_{i2}b^{1-\alpha}]^{\frac{1}{1-\alpha}}$.

¹⁷This is the missing link between trade and specialization in Kose and Yi(2001).

where we use the GDP for output, the stock of capital we constructed using series of investment and we use the total number of hours worked as a measure of labor.

The last step is to effectively estimate the productivity shock process:

$$\begin{pmatrix} z_{1t+1} \\ z_{2t+1} \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} * \begin{pmatrix} z_{1t} \\ z_{2t} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{pmatrix}$$

where $\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix} \ \tilde{} N(0, \Sigma).$

Estimates for elements of A and Σ are presented in *Table 3*. Numbers in parentheses represent standard errors.

[Table 3]

We have now all the ingredients needed to compute the steady state, to log-linearize the system around this steady state and then to apply the King Plosser Rebelo (1987) procedure.

5 Results

This section reports the average results across 1000 stochastic simulations. One period in the model corresponds to one quarter in the data. Therefore, for the case of USA-Mexico we simulate the economy over 72 periods. In *Table 4* we present the result of these simulations and compare them with what we observed in data.

In the first exercise we made, whose results we present in *Table 4*, we consider a decrease in trade costs from 45% to 15%. *Table 5* shows what happens with the predictions for a bigger fall in the trade costs.

[Table 4]

Comparing our results to the data we can see that the model performs well in predicting the correlation in consumption: a jump from -0.18 to 0.27 in the model corresponds to the jump observed in the data, from -0.28 before NAFTA to 0.32 after NAFTA. However, in the data the increase is higher than what our model predicts. In terms of investment, the predicted jump is far smaller than what we found in the empirical study. Moreover, the initial level of correlation is bellow the data. In the case of output correlation, we observe the same smaller jump in the predictions than in the data. However, for all the correlations, the direction of changes coincides with the one found in the data: all the correlations increase.

Moreover, *Figures 4-6* show that the pattern followed by the model predictions has the same U-shape as the data, although the curvature is much softer than in the data.

[Figures 4-6]

Table 5 presents the results of our simulation for a bigger fall in the trade cost, τ . We consider the extreme case where before the trade agreement there is almost no trade between countries (this corresponds to a trade cost $\tau = 0.95$), then, when the trade agreement is signed, the trade cost becomes very close to 0 ($\tau = 0.05$). On the one hand, the results show that all the correlations are very sensitive to changes in trade cost. On the other, all the predictions

after NAFTA are relatively far bellow what we observe in the data. This is an evidence that, although a reduction in the trade costs is an important feature of a trade agreement, it is not a perfect proxy for all the changes a trade agreement means.

[Table 5]

Up to now we saw that under complete financial markets a decrease in the trade cost in the international business cycle model is able to reproduce the direction of change in comovements but quantitatively the results fall short from the data.

Table 6 presents the results provided by the model under the incomplete financial markets specifications: bond economy and financial autarky. As in the previous part of the paper, we are investigating how sensitive the correlations are to changes in trade cost. The results presented in the following table correspond to the same change in the trade cost we used in the benchmark experiment ($\tau = 0.45$ before NAFTA and $\tau = 0.15$ after signing NAFTA). The results are compared not only to the data, but also to the complete markets model's predictions.

As we can observe, the predictions of the model under the bond market specification are relatively similar to the case of complete markets: the predicted correlations in consumption are very close to the empirical findings, but we cannot say the same about investment and output. However, the bond market simulation approximates better the pre-NAFTA correlation in consumption while the post-NAFTA correlation is better approximated under the complete market case.

[Table 6]

Under the specification of financial autarky we can observe a clear increase in the correlations of consumption and output. However, the increase is too big and pushes the predictions of the model too far from the data. When we analyze the investment correlation, once again all three specifications seem to lead to predictions relatively far from the real data.

The conclusion of these experiments is that if we combine the predictions of the bond economy in the pre-NAFTA period with the complete market predictions post-NAFTA we observe that the results get closer to the behavior observed in the data. This could be evidence of the fact that NAFTA meant not only a reduction of trade cost, but also better risk hedging opportunities for the countries involved.

6 Conclusions and future research

The empirical evidence we have shows that the correlations in consumption, investment and output between Mexico and the US increased after 1992, when the preparatives for signing the North American Free Trade Agreement finished. We interpret these results as an evidence of NAFTA's impact on the US-Mexico comovements.

We have shown that under different financial markets specifications, the model provides good qualitative results: a reduction in the trade barriers produces an increase in the comovements in all the three macroeconomic variables (consumption, investment and output). On the other hand, all three specifications of the model fail to give the quantitative increase in comovements we observe in the data. Our results show that the bond economy performs better in terms of matching the data in the pre-NAFTA period, while the complete markets model offers the best fit for the post-NAFTA period. This suggests that NAFTA meant not only a reduction of trade cost but also better financial integration for the three countries involved.

We therefore think it would be interesting for future research to find empirical evidence of Mexico's financial integration after signing the North American Free Trade Agreement. It is our belief that, enriching the international business cycle model so that to account not only for reduction in the cost of trade but also for the financial liberalization would provide predictions that are closer to the empirical evidence.

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8 Tables and Figures

Corr / NAFTA	Before	After
corr (c ₁ , c ₂)	-0.28	0.32
corr (i ₁ , i ₂)	-0.40	0.58
corr (y ₁ , y ₂)	-0.19	0.56

TABLE 1. Mexico – USA: Empirical evidence

Source: OECD Quarterly National Accounts Note: c = consumption, i = investment, y = output. All the variables are Hodrick-Prescott-filtered with a smoothing parameter of 1600. The periods "before" and "after" corresponds to the following quarters: "before": 1985:1 – 1993:4, "after": 1994:1 – 2002:4

TABLE 2. Parameter Values

Туре	Name	Symbol y value
	Discount Factor*	$\beta = 0.99$
Preferences	Consumption share	$\begin{cases} \mu_{USA} = 0.34 \\ \mu_{MEX} = 0.38 \end{cases}$
	Curvature parameter *	$\gamma = 2$
	Capital share	$\theta = 0.36$
Technology	Depreciation rate	$\begin{cases} \delta_{\scriptscriptstyle USA} = 0.025 \\ \delta_{\scriptscriptstyle MEX} = 0.028 \end{cases}$
* Imported from BKK	TABLE 3. Productivity sh	ock
Productivity transit	· · · ·	$A = \begin{bmatrix} 0.951 & -0.011 \\ 0.023 & 0.006 \\ 0.045 & 0.939 \\ 0.020 & 0.025 \end{bmatrix}$
Std. dev. of innovations to productivity		$\begin{cases} \sigma_{\varepsilon 1} = 0.0089 \\ \sigma_{\varepsilon 2} = 0.0976 \end{cases}$

Correlation between innovations to productivity	$corr(\varepsilon_1, \varepsilon_2) = 0.276$
---	--

	TABLE 4. Benchmark experiment		
Mexico - USA	Before	After	
	Da	nta	
$\operatorname{corr}\left(c_{1},c_{2} ight)$	-0.28	0.32	
corr (i ₁ , i ₂)	-0.40	0.58	
corr (y ₁ , y ₂)	-0.19	0.56	
	Mo	odel	
corr (c ₁ , c ₂)	-0.18	0.27	
corr (i ₁ , i ₂)	-0.72	-0.68	
corr (y ₁ , y ₂)	0.21	0.35	

Note: c = consumption, i = investment, y = output. All the variables are Hodrick-Prescott-filtered with a smoothing parameter of 1600. The periods "before" and "after" corresponds to 1985:1 – 1993:4, respectively 1994:1 – 2002:4.

	ivity analysis		
Mexico - USA	Before	After	
	Benchman	rk experiment	
corr (c ₁ , c ₂)	-0.18	0.27	
corr (i ₁ , i ₂)	-0.72	-0.68	
corr (y ₁ , y ₂)	0.21	0.35	
	Huge fall in trade cost		
corr (c ₁ , c ₂)	-0.93	0.28	
corr (i ₁ , i ₂)	-0.90	-0.65	
corr (y ₁ , y ₂)	-0.01	0.30	

Note: c = consumption, i = investment, y = output. All the variables are Hodrick-Prescott-filtered with a smoothing parameter of 1600. The periods "before" and "after" corresponds to 1985:1 – 1993:4, respectively 1994:1 – 2002:4.

correlations	corr (c ₁ , c ₂)	corr ((i ₁ , i ₂)	corr (y ₁ , y ₂)
period	before	after	before	after	before	after
γ=0.75	-0.54	-0.25	-0.23	0.00	0.31	0.49
γ=1	-0.44	0.14	-0.38	-0.20	0.27	0.43
γ=1.25	-0.28	0.38	-0.51	-0.36	0.25	0.40
γ=1.5	-0.20	0.42	-0.60	-0.51	0.22	0.37
γ=1.75	-0.19	0.35	-0.66	-0.59	0.23	0.36
$\gamma=2$ (Benchmark exp)	-0.18	0.28	-0.72	-0.67	0.21	0.34
γ=3	-0.19	-0.01	-0.84	-0.83	0.20	0.31
γ=4	-0.18	-0.14	-0.90	-0.88	0.19	0.30
Data	-0.28	0.32	-0.40	0.58	-0.19	0.56

TABLE 6. γ-Sensitivity analysis

Note: c = consumption, i = investment, y = output. All the variables are Hodrick-Prescott-filtered with a smoothing parameter of 1600. The periods "before" and "after" corresponds to 1985:1 – 1993:4, respectively 1994:1 – 2002:4.

Corr / NAFTA	Before	After	
	Data		
corr (c ₁ , c ₂)	-0.28	0.32	
corr (i ₁ , i ₂)	-0.40	0.58	
corr (y ₁ , y ₂)	-0.19	0.56	
	Complete	markets	
corr (c ₁ , c ₂)	-0.18	0.27	
corr (i ₁ , i ₂)	-0.72	-0.68	
corr (y ₁ , y ₂)	0.21	0.35	
	Bond economy		
corr (c ₁ , c ₂)	-0.26	0.22	
corr (i ₁ , i ₂)	-0.87	-0.71	
corr (y ₁ , y ₂)	0.48	0.89	
	Financial autarky		
corr (c ₁ , c ₂)	-0.11	0.69	
corr (i ₁ , i ₂)	-0.97 -0.89		
corr (y ₁ , y ₂)	0.94	0.94	

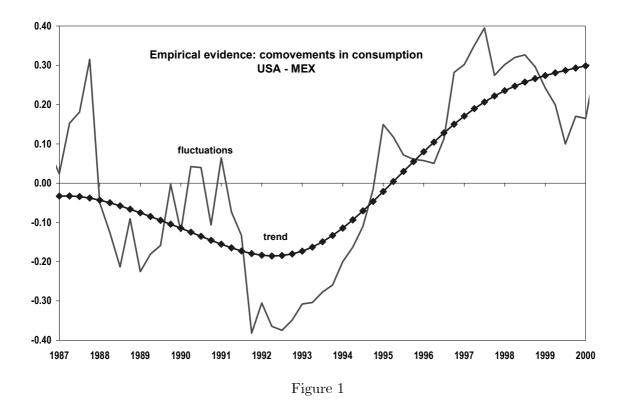
TABLE 7. Incomplete financial markets

 Note:
 c = consumption, i = investment, y = output. All the variables are Hodrick-Prescott-filtered with a smoothing parameter of 1600. 1 period in the simulated results corresponds to 1 quarter in data. The periods "before" and "after" corresponds to the following quarters:

 México – USA:
 1985:1 – 1993:4, 1994:1 – 2002:4

 Canada – Mexico:
 1987:1 – 1993:4, 1994:1 – 2000:4

 USA – Canada:
 1987:1 – 1993:4, 1994:1 – 2000:4



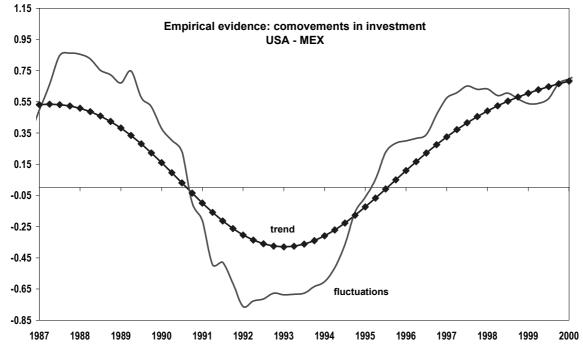


Figure 2

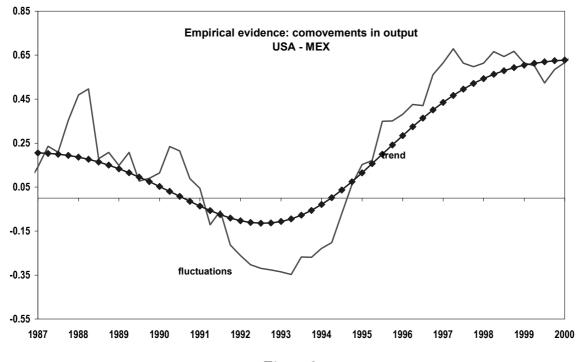


Figure 3

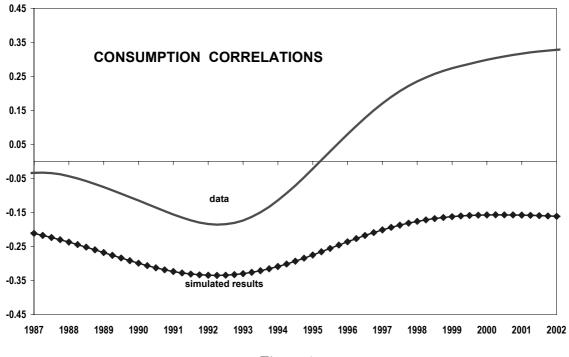


Figure 4

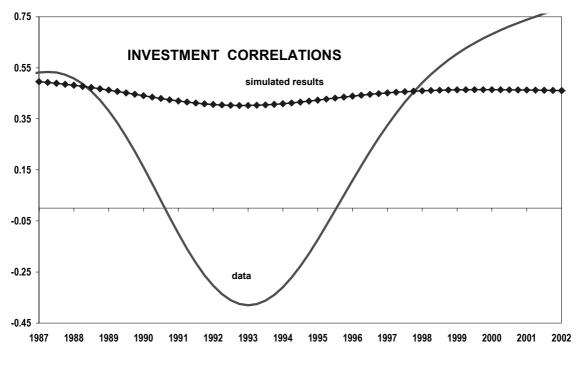


Figure 5

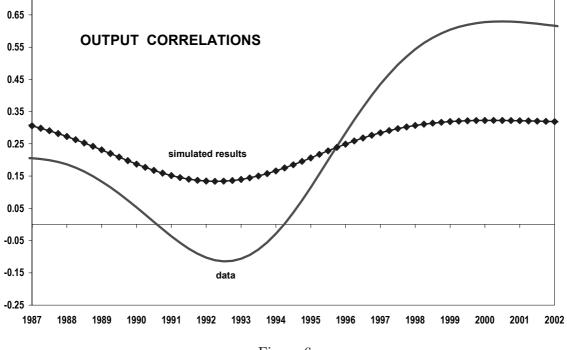


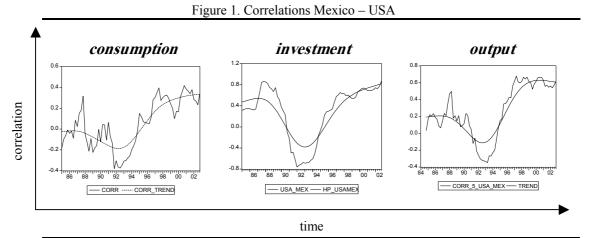
Figure 6

9 Appendix 1: Data description

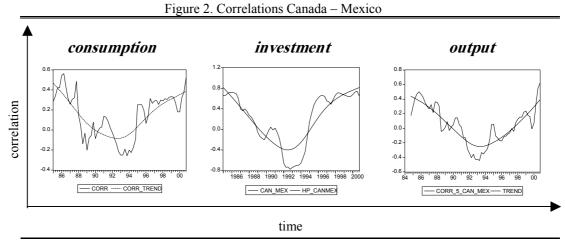
Data used to compute correlations in consumption, investment and output is taken from OECD Quarterly National Account and International Monetary Fund (International Financial Statistics). From OECD, Main Economic Indicators we took quarterly data for weekly hours of work and we use it to calculate a proxy for labor used in production for the case of United States. For Mexico the data I use comes basically from INEGI and it was provided to me by Felipe Meza.

Consumption in this model is computed as the sum between private and government consumptions and investment like gross fixed capital formation plus changes in inventories. Then, in order to compute the stock of capital we take the quarterly depreciation rate equal to 2.5%.

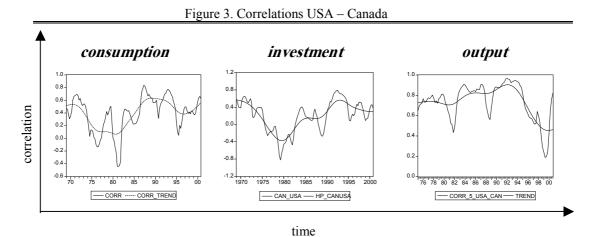
10 Appendix 2: Empirical evidence



Note: Correlations are computed over a 5-year moving window. Smooth line represents the HP trend.



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Note: Correlations are computed over a 5-year moving window. Smooth line represents the HP trend.

TA	ABLE 1. Empirical eviden	ce	
Corr / NAFTA	Before	After	
	Mexico – USA		
$\operatorname{corr}(\mathbf{c}_1,\mathbf{c}_2)$	-0.28 0.32		
corr (i ₁ , i ₂)	-0.40	0.58	
corr (y ₁ , y ₂)	-0.19	0.56	
	Mexico – Canada		
$corr(c_1, c_2)$	-0.14	0.43	
corr (i ₁ , i ₂)	-0.55	0.71	
corr (y ₁ , y ₂)	-0.38	0.34	
	USA –	Canada	
$\operatorname{corr}(\operatorname{c}_1,\operatorname{c}_2)$	0.71	0.72	
corr (i ₁ , i ₂)	0.38 0.46		
corr (y ₁ , y ₂)	0.92 0.47		

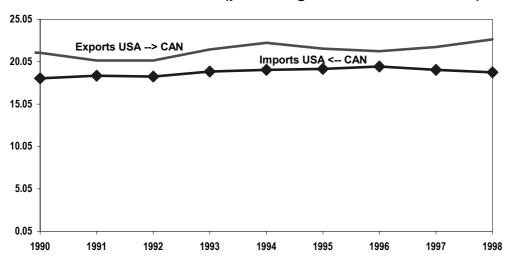
Source: OECD Quarterly National Accounts

Source: OECD Quarterly National Accounts
Note: c = consumption, i = investment, y = output. All the variables are Hodrick-Prescott-filtered with a smoothing parameter of 1600.
The periods "before" and "after" corresponds to the following quarters: México – USA: 1985:1 – 1993:4, 1994:1 – 2002:4 Canada – Mexico: 1987:1 – 1993:4, 1994:1 – 2000:4 USA – Canada: 1987:1 – 1993:4, 1994:1 – 2000:4

11 Appendix 3: Trade data

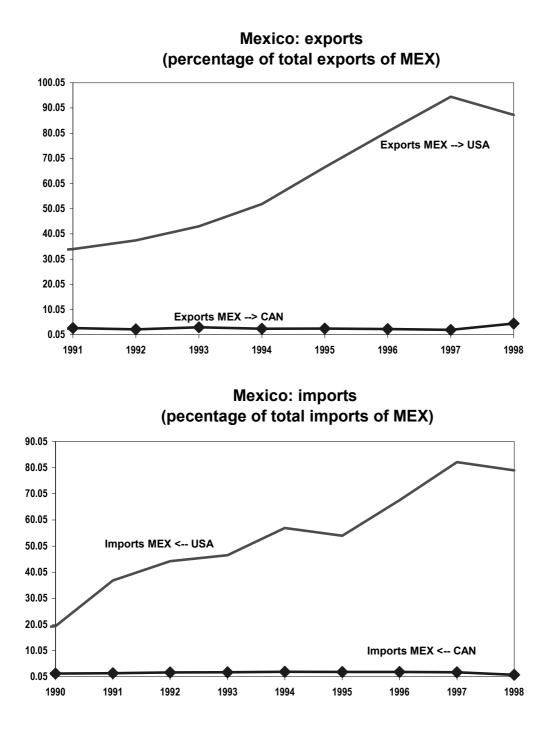
This appendix contains comparative trade statistics for the US, Mexico and Canada. The data used is from Krueger (1999).

The next graph shows how in 1994 the percentage of trade USA-CAN of the total trade of USA decreased instead of increase.



Trade USA -- CAN (percentage of total trade of USA)

The next graphs represent a comparative study in order to highlight the difference between the importance of the US and Canada in Mexico's trade. The exports/imports of MEX to/from USA and CAN are represented as the percentage of total exports/imports of Mexico.



12 Appendix 4: Calibration for Armington aggregator

This appendix describe in detail how we calibrate the parameters for Armington aggregator: the elasticity of substitution between intermediate goods and the weight every country give to its domestically produced good.

• Elasticity of substitution between intermediate goods: α from Armington aggregator:

From first order conditions for intermediate goods we impose the condition that the sum of Armington aggregator to be 1:

$$(1 - \omega_1)a_1^{-\alpha}G_1^{\alpha} = q_1^a \Longrightarrow \omega_1 = 1 - q_1^a (\frac{a_1}{G_1})^{\alpha}$$
$$\omega_1(1 - \tau)^{1-\alpha}b_1^{-\alpha}G_1^{\alpha} = q_1^b \Longrightarrow \omega_1 = q_1^b (\frac{b_1}{G_1})^{\alpha}(1 - \tau)^{\alpha - 1}$$
$$(1 - \omega_2)b_2^{-\alpha}G_2^{\alpha} = q_2^b \Longrightarrow \omega_2 = 1 - q_2^b (\frac{b_2}{G_2})^{\alpha}$$
$$\omega_2(1 - \tau)^{1-\alpha}a_2^{-\alpha}G_2^{\alpha} = q_2^a \Longrightarrow \omega_2 = q_2^a (\frac{a_2}{G_2})^{\alpha}(1 - \tau)^{\alpha - 1}$$

Taking into account that in steady state we have constant share of imports in output, we can compute

$$\frac{a_1}{G_1} = \frac{1-s_1}{q_1^a} \text{ and } \frac{b_1}{G_1} = \frac{s_1}{q_1^b}, \frac{a_2}{G_2} = \frac{s_2}{q_2^a} \text{ and } \frac{b_2}{G_2} = \frac{1-s_2}{q_2^b}$$

Therefore

• – for the first country we have a coefficient α satisfying

$$1 - q_1^a (\frac{1 - s_1}{q_1^a})^\alpha = q_1^b (\frac{s_1}{q_1^a})^\alpha (1 - \tau)^{\alpha - 1}$$

- for the second country we have a coefficient α satisfying

$$1 - q_2^b (\frac{1 - s_2}{q_2^b})^\alpha = q_2^a (\frac{s_2}{q_2^b})^\alpha (1 - \tau)^{\alpha - 1}$$

Therefore we have in both countries α like a function of τ .

• Coefficients for Armington aggregator (weights):

The functional form we use for Armington aggregator is

$$G(a_i, b_i) = ((1 - \omega)a_i^{1 - \alpha} + \omega(1 - \tau)^{1 - \alpha}b_i^{1 - \alpha})^{\frac{1}{1 - \alpha}}$$

In order to calibrate the coefficient ω we start from the imports share of output, ratio calibrated with the existing data. Therefore, if we denote this share by s we have that

$$s_1 = \frac{q_1^b b_1}{p_1 G(a_1, b_1)}, s_2 = \frac{q_2^a a_2}{p_2 G(b_2, a_2)}$$

Since we normalized both prices of final goods with 1, we have that $p_1 = p_2 = 1$.

We will compute the coefficient only for the first economy since for the second one will be completely symmetric taking into account the symmetry of our system. From the cost minimization problem in the final good production, we have:

$$\min(q_1^a a_1 + q_1^b b_1)$$

s.t. $(1 - \omega)a_1^{1-\alpha} + \omega(1 - \tau)^{1-\alpha}b_1^{1-\alpha} = G_1^{1-\alpha}$

First order conditions:

$$\begin{array}{c} q_1^a = \lambda (1-\omega)(1-\alpha)a_1^{-\alpha} \\ q_1^b = \lambda \omega (1-\alpha)(1-\tau)^{1-\alpha}b_1^{-\alpha} \\ \end{array} \right\} \Longrightarrow (\frac{b_1}{a_1})^{\alpha} = \frac{q_1^a}{q_1^b} \frac{\omega (1-\tau)^{1-\alpha}}{1-\omega} \Longrightarrow \\ \Longrightarrow \frac{b_1}{a_1} = (\frac{q_1^a}{q_1^b} \frac{\omega (1-\tau)^{1-\alpha}}{1-\omega})^{1/\alpha} \end{array}$$

Now, from the share of imports we will get the coefficients:

$$\begin{split} s_1 &= \frac{q_1^h b_1}{p_1 G(a_1, b_1)} \\ p_1 G(a_1, b_1) &= q_1^a a_1 + q_1^h b_1 \\ s_1 &= \frac{1}{\frac{q_1^a}{q_1^a} \frac{a_1}{a_1} + 1} = \frac{1}{\frac{q_1^a}{q_1^a} (\frac{q_1^a}{q_1^a} \frac{\omega(1-\tau)^{1-\alpha}}{1-\omega})^{-1/\alpha} + 1} \Rightarrow \\ &\Rightarrow \frac{\omega}{1-\omega} = [(\frac{1}{s_1} - 1) * (\frac{q_1^a(1-\tau)}{q_1^h})^{\frac{1-\alpha}{\alpha}}]^{-\alpha} \\ &\Rightarrow \frac{\omega}{1-(\omega)} = [(\frac{1}{s_1} - 1) * (\frac{q_1^a(1-\tau)}{q_1^h})^{\frac{1-\alpha}{\alpha}}]^{-\alpha} \\ &\Rightarrow \frac{([(\frac{1}{s_1} - 1) * (\frac{q_1^a(1-\tau)}{q_1^h})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}}{1 + [(\frac{1}{s_1} - 1) * (\frac{q_2^b(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}} = \frac{[(1-s_1) * (\frac{q_1^a(1-\tau)}{q_1^h})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}}{s_1 + [(1-s_1) * (\frac{q_1^a(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}} \text{ in the first country} \\ \omega_2 &= \frac{[(\frac{1}{s_2} - 1) * (\frac{q_2^b(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}}{1 + [(\frac{1}{s_2} - 1) * (\frac{q_2^b(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}} = \frac{[(1-s_2) * (\frac{q_2^b(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}}{s_2 + [(1-s_2) * (\frac{q_2^b(1-\tau)}{q_2^a})^{\frac{1-\alpha}{\alpha}}]^{-\alpha}} \text{ in the second country} \end{split}$$