Sale of Information by an Informed Trader

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Abstract

This paper analyses the effect of the sale of information by an informed strategic trader (seller) to another strategic player (buyer). It shows that the seller will never fully divulge his information; he will sell a noisy signal of his information to the other player. If they trade for longer periods, the seller will, moreover, try to "hide" his information from the other trader by trading less aggressively on it. It also shows that in a one period model prices are more efficient in a setting with a sale of information than with just one informed player - this even though the seller trades less on his true information.

Keywords: Market Microstructure, Sale of Information

JEL Codes: G1, D82

1 Introduction

Strategic traders participate in the market for an asset if they believe they have informational advantage over the market makers (who set the price at the asset's expected value given their information). If they had only as much information as the market maker they would have no incentive to trade - the price that they expect the market maker to set would be exactly what they expect the value of the asset to be. Information is crucial for the strategic traders to make any profits and they would be willing to pay for it. Given this scenario, the questions this paper seeks to address are: Will an informed large risk neutral trader be willing to sell his information to another large risk neutral trader and under what conditions will he be willing to do so? If he does sell his information what kinds of trading patterns would that give rise to? And what is the impact of the sale on the efficiency of prices?

Admati and Pfleiderer (1988) show that under direct sale of information, a risk neutral trader would never share his informational advantage though a risk averse trader might . Here we claim that there can still be sale of information by a risk neutral trader *if* he sells a noisy signal of his information. If the trader is risk-neutral and he sells exactly his information then there are two traders in the market who both have exactly the same information and know of the existence and the information of the other. One way to look at it is that the market moves from a monopoly to an oligopoly with the two traders competing as in a Cournot game. The competition leads to lower total profits for the two traders combined than the monopolist's profits alone. So the informed trader is always better off being a monopolist than selling his information. On the other hand with risk averse traders there is the benefit of better risk sharing which makes up for the loss in aggregate profits. This paper shows that sale of information can be profitable even for a risk neutral trader if he is allowed to sell a "noisy" signal of his information. This makes intuitive sense because now the informed trader retains an informational advantage over both the market maker and the other player – he knows the true value as well as the other players signal. Since he knows exactly how much the prices will be affected by the trade of the other player in equilibrium, he can use it to his advantage. By making the signal noisy enough, he will make larger expected profits than in the monopoly case. As for the second trader it is always better to buy some information. Even with the noisy signals he makes positive expected profits. Any informational advantage over the market maker makes trade possible and gives him positive expected payoffs.

We look for a linear equilibrium and find that the seller's strategy is linear in his information and also in the signal sold by him. There is an asymmetry arising in the model due to the fact that the seller knows the true value and also the information of the buyer whereas the buyer only knows the signal he bought. Since the strategies are linear, with positive probability the seller's true information can get swamped by the signal. So we could observe both traders buying the asset even when the true value was lower than the expected price.

In longer horizons the information buyer would not only have the advantage of the original signal he bought, he would also have the ability to learn more than the market maker from the trade of the information seller. He can update his information each period. In the particular model examined here he updates to a linear combination of his prior and the noisy signal he extracts from the information seller's trade. This might make the information seller more reluctant to trade on his information in the earlier periods. He might try to 'hide' his information by trading less aggressively on it. In this paper we see that under some conditions he trades less on his information. While trading on his true information he has to take into account the fact that the market maker and the buyer both learn from his trade. The buyer, because his prior is superior to that of the market maker, extracts a more precise signal from the seller's trade than the market maker. The seller has to take into account both these 'learnings' when he decides the level of aggression to choose.

Another thing to consider is the informational efficiency of prices or the extent to which the private information is revealed by prices. With only one perfectly informed trader, prices reveal one half of the information of the trader. In this model there is now another trader with a noisy signal, which results in a larger proportion of the market having some information. In fact, with low levels of noise in the signal the average level of information of the 3 players (buyer, seller and market maker) is higher than in the case with only one trader. This makes prices more efficient even though the seller might have incentives to trade less on his information. The prices are in fact more efficient than in the single informed trader case.

The structure of the model used in the paper is based on Kyle (1985). He considers a model with one informed trader (knows the liquidation value), a market maker and noise traders. They trade in the asset for some periods at the end of which the liquidation value is announced and profits are realised. Market makers are assumed to set prices equal to the expected value of the asset given their information. In a linear equilibrium prices are linear in the total trade and the insider's trades is linear in the liquidation value. In the current model the same hold except for the fact that the seller's strategy is linear in the buyer's signal as well.

The following papers consider the sale of information. Admati and Pfleiderer (1986) consider a case when the seller does not trade and has the option to sell to a fraction from a continuum risk averse traders. Since the seller is a monopolist his profit is the sum of the profits of all traders. They consider two ways of selling information - sell the same information to everyone or sell iid signals. They find that in a rational expectations equilibrium it is never optimal to let a positive fraction of the traders know the exact value of the asset - this would lead to the information being exactly reflected in the prices and zero trading profits. So as in the current model he always sells a noisy signal.They also find it is strictly better to sell iid signals than to sell everyone exactly the same signal value.

In Admati and Pfleiderer (1988) they find information selling profitable for a risk averse seller as it is a means of sharing risk. They also find that indirect sale of information (through a managed mutual fund) is more profitable means of selling information. This is so because the expected total trading profit is decreasing in the number of traders. With a mutual fund this problem is resolved as there is only one trader. In Admati and Pfleiderer (1990) they show that indirect sale of information is profitable if investors in a mutual fund can be charged a fixed fee and a fee per share. But in this paper they assume the information seller cannot trade on his information.

Jordi Caballe (1993) considers a model where a seller of information can either sell the information or use it to trade. There is a cost of producing information. The sum of profits is maximised when there is one trader and so if the sale takes place, information is sold to one trader only and the seller of information extracts all the surplus. With a cost of production of information the seller adds an optimal level of noise to the information.

Fishman and Hagerty (1995) consider a model where there is more than one informed trader, one of whom can commit to selling his information to a number of uninformed traders. Selling information is shown to be equivalent to committing to trade aggressively. If a trader commits to trading aggressively, the other traders trade less aggressively and the aggressive trader takes away a larger share of the total profits.

We do not address the issue of credibility here; it has been studied by Allen (1990). He finds that markets for financial information can operate even if they have a credibility problem. Brennan and Chordia (1991) study the case where the seller of information is risk neutral but the buyers are risk averse. The most efficient way to charge should depend on the signal realisation but if the signal is not contractible then the alternative is brokerage commissions. Bushman and Indjejikian (1995) consider the slightly different problem of the impact of public disclosures on the profits of insiders. A public disclosure takes away some of the insiders knowledge edge but it ends up increasing his profits by making other less informed traders less aggressive.

The next section describes the model. Section 3 looks at the problem when there is only one trading period and section 4 extends to the case with two trading periods. Sections 5 and 6 explain some of the interesting trading patterns in the model with two trading periods. Section 7 looks at the efficiency of prices.

2 The Model

The basic setup of the model follows Kyle (1985). There are two assets in the economy – a risk free asset and a risky asset. The interest rate on the risk free asset is normalized to zero. The risky asset has a random liquidation value which is denoted by v, where v is distributed normally as $N(\mu_v, \sigma_v^2)$. The liquidation value is realized at the beginning of the game but at this stage it is not observable to all the players. Its value is announced to all the players at the end of game.

There are three kinds of traders in this economy – risk-neutral strategic traders, noise traders and market makers. In the particular problem that we analyse there are two distinct risk-neutral strategic traders – one of them is informed and is willing to sell his information (the seller) and the other one is willing to buy the information (the buyer). The seller has the advantage of being the only player in the economy to know the true realization of the liquidation value v before trading begins. He is also willing to sell this information, or a noisy version of it, to the buyer for a price before trading commences. Without any information, the buyer will have to stay out of the trading and get zero expected profits. He is, therefore, willing to pay up to his expected trading profits for any information. If the seller decides to sell the information, he sells a signal $s = v + \varepsilon$ where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. If $\sigma_\varepsilon^2 = 0$ then the seller is giving away exactly his value otherwise he sells him a noisy signal of the true valuation. If the sale of information takes place then both buyer and seller trade strategically for two periods.

The noise traders are non-strategic traders and they trade for reasons other than to make a profit. For instance they might have liquidity constraints which force them to trade. We do not model their trading behaviour, we just assume they trade a random amount each period i, u_i, where $u \sim N(0, \sigma_u^2)$. We also assume u_i is independently drawn each period and it is independent of any other random variable in the model.

Trade by the strategic and noise traders takes the form of market orders - where the traders submit net buy orders which they are willing to execute at any price set by the market maker. The market maker sets up an equilibrium pricing rule and he is then willing to absorb all trades given the pricing rule. The market maker observes only the aggregate net buy order and not the individual net orders. We also assume that the market maker sets prices equal to the risky asset's expected liquidation value given his information. The assumption can be motivated by the presence of Bertrand completion (not modeled) amongst market makers, which would mean that they make zero expected profits.

In this model the seller of information sells his information directly to the buyer. The buyer then uses this information to trade strategically. The setting can be motivated by thinking of an expert selling his knowledge/views on a trade magazine or financial newsletter and charging a fee for doing so. Of course, there are issues of credibility – why should the buyer buy the information if there is no verification of the quality of this information? We sidestep this issue here by assuming that the seller has reputational considerations and he always sells his information at the quality promised. There is also the concern that the buyer might have incentives to resell the information – we assume he gets his information just before the start of trading and doesn't have the time to resell.

The sequence of events is as follows:

Period 0: The seller receives accurate information ,v , regarding the liquidation value of the stock. He then decides whether or not to sell some of his information to the buyer in the form $s = v + \varepsilon$. If he does sell it, he also has to decide on the level of noise to add - he needs to choose σ_ε^2. Since the buyer is risk neutral, the seller will be able to extract the buyer's expected profits from the information as the fee for the information. The seller's expected profit is the sum of his expected trading profits and the fee for information (the buyer's expected trading profit).

Period 1 : The traders then submit market orders for period 1(the seller submits x_{S1} and the buyer submits x_{B1}) given the equilibrium pricing rule of the market maker. The market maker sets prices for period one, p_1 . The buyer then extracts from the total trade a noisy signal of the seller's trade (which would depend on the true liquidation value and the buyers signal). Using this signal, the buyer updates his beliefs to s_2.

Period 2 : The traders again place market orders for the last period where the buyer's order is now a function of his updated signal (the seller submits x_{S2} and the buyer submits x_{B2}). The market maker sets the prices, p_2 , and the market clears.

Period 3 : The true liquidation value is announced and all traders liquidate their positions and realise profits.

A sequentially rational Bayes Nash Equilibrium is given by strategy proflie $\{x_{S1}^*, x_{B1}^*, x_{S2}^*, x_{B2}^*, p_{1,}p_2\}$ and the beliefs of the buyer $\{s_1, s_2\}$such that :

(1)

$$
\begin{aligned}
x_{S2}^{*} &\in \arg\max_{x_{s2}^{*}} \pi_{S2} = E\left((v-p_2)\, x_{S2}\right) \\
x_{B2}^{*} &\in \arg\max_{x_{B2}^{*}} \pi_{B2} = E\left((v-p_2)\, x_{B2}\right)
\end{aligned}
$$

(2)

$$
\begin{aligned}
x_{S1}^{*} &\in \arg\max_{x_{S1}^{*}} \pi_{S1} = E\left((v-p_1)\, x_{S1} + \pi_{S2}(x_{S2}^{*})\right) \\
x_{B1}^{*} &\in \arg\max_{x_{B1}^{*}} \pi_{B1} = E\left((v-p_1)\, x_{B1} + \pi_{B2}\left(x_{B2}^{*}\right)\right)
\end{aligned}
$$

(3)

$$
\begin{aligned}
p_1 &= E(v|x_{S1}^{*} + x_{B1}^{*} + u_1) \\
p_2 &= E(v|x_{S1}^{*} + x_{B1}^{*} + u_1, x_{S2}^{*} + x_{B2}^{*} + u_2)
\end{aligned}
$$

(4)

$$
\begin{aligned}
s_1 &= E(v|s) = q\mu_v + (1-q)s \text{ } where \text{ } q = \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2} \\
s_2 &= E(v|s, x_{S1}^{*} + u_1)
\end{aligned}
$$

Equation (1) is the profit maximizing condition for the last period for the two strategic players. Equation (2) is the profit maximizing condition in the first trading period given their optimal choices in the last period. Equation (3) states that the market maker will set prices equal to his expectation of the liquidation value given his information each period. The last equation, (4), is the updating rule for the buyer. If he buys a noisy signal then he updates his signal at the end of the first round of trading to s_2, which is his expectation of the liquidation value given his original signal and the information he extracts from trading (which is a noisy signal of the seller's trade).

We look for a linear equilibrium in pure strategies. The buyer and seller can trade only if they have more information or if their valuation of the asset is different from that the market maker. Their informational advantage can be expressed by the difference in a trader's expectation and the market makers expectation.The trading strategy for the seller will be linear in his informational advantage (the true value of the asset less the market maker's expectation) and also in the informational advantage of the buyer (the buyer's beliefs less the market maker's expectation). The coefficient of the seller's own informational advantage is aggressiveness - it measures how strongly he is willing to trade on his information in that period. Similarly the buyer's strategy will be a product of his trading intensity (aggression) and his informational advantage. With both informed traders following a linear strategy and all random variables being normal, we can apply the Projection Theorem for Normal Variables and get equilibrium prices linear in total trade. Prices will be of the form $p_i =$

$\mu_v + \lambda_i(x_{Si} + x_{Bi} + u_i)$ where λ_i is the reciprocal of market depth. It measures the impact of a one unit increase in total trade on prices. The lower λ_i is, the more liquid is the market.

3 One Trading Period Problem

We first consider the problem where there is only one trading period. The seller of information chooses whether or not to sell information before trading commences. If he chooses to sell, he has to decide on the optimal level of noise in the signal. In the trading period, all traders place their market orders. After receiving the market orders, the market maker sets the price and markets clear. In the next period the true liquidation value is announced and the profits are realised.

We compare 3 possible scenarios: one is the standard Kyle (1985) model with one monopolistic informed trader (this is the case with no sale of information), the second is the one studied by Admati and Pfleiderer (1988) where the seller sells exactly his signal and the third is the case where the seller sells a noisy signal. We first present the linear pure strategies equilibria in the three cases. The linear equilibrium will have prices, p, and trade, x, of the form:

$$p = \mu_v + \lambda(trade)$$

and the strategic trader will choose a net buy order, x, to maximize trading profits:

$$x = \arg\max_x E(v - p)x$$

3.1 One Informed Trader

The one informed trader case was analysed by Kyle (1985). We present the results here. In this case, the strategic trader has monopoly powers over his information. His trade, x^1, is proportional to the difference between his valuation and the market's expected value. The trader's equilibrium trading strategy (x^1) and the depth parameter (λ_1) for the pricing rule are given by:

$$x^1 = \frac{v - \mu_v}{2\lambda_1}; \lambda_1 = \frac{\sigma_v}{2\sigma_u}$$

The second order condition ensures $\lambda_1 > 0$. Using the previous results we can derive the expected profits to the trader :

$$E(\pi^1) = \frac{1}{\lambda_1} E\left(\frac{v - \mu_v}{2}\right)^2 = \frac{2\sigma_u}{\sigma_v}\left(\frac{v - \mu_v}{2}\right)^2$$

This case will serve as the benchmark case. The seller of information in our model can always choose not to sell information and revert to this equilibrium.

3.2 Two Symmetrically Informed Traders

This is the case analysed by Admati and Pfleiderer (1988) where the seller sells exactly his information. In a way, the market turns into a duopolistic market instead of the seller retaining his monopoly. Since the buyer is risk neutral, the seller will be able to extract all of the buyer's expected profits as a fee for the information.

After the information is sold, the two traders will be symmetric and will be using the same trading strategies. The equilibrium strategies of the seller (x_s^2), the buyer (x_B^2) and the market maker (λ_2) are given by:

$$x_S^2 = x_B^2 = \frac{v - \mu_v}{3\lambda}; \lambda_2 = \frac{\sqrt[2]{2}\sigma_v}{3\sigma_u}$$

where the second order condition ensures $\lambda_2 > 0$. And this leads to expected profits of:

$$E(\pi_S^2) = E(\pi_B^2) = \frac{1}{\lambda_2}\left(\frac{v - \mu_v}{3}\right)^2 = \frac{3\sigma_u}{\sqrt[2]{2}\sigma_v}\left(\frac{v - \mu_v}{3}\right)^2$$

By selling his all his information, the seller looses his informational edge over the buyer. The competition amongst these two traders with the same information leads to a fall in the sum of their profits as compared to the monopolistic case. The seller of information, then, has no incentive to lose his monopoly by selling his information.

3.3 Two Asymmetrically Informed Traders

The seller in this case sells $s = v + \varepsilon$ where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ and $\sigma_\varepsilon^2 > 0$ to differentiate this case from the previous case. In this case where the buyer receives only a noisy signal of the true value, the seller still retains an informational edge over the buyer as well as the market maker. He still has some monopoly powers and he also gets to extract a fee for his information. The seller's trading strategy will be different from the previous cases; it will now be linear in both his information and also in the information he sold to the buyer. The buyer's trading strategy will be linear in his information only.

The strategies are given by:

$$\begin{aligned} x_S^3 &= \frac{v - \mu_v}{2\lambda_3} - \frac{s - \mu_v}{6\lambda_3} \\ x_B^3 &= \frac{s - \mu_v}{3\lambda_3} \end{aligned}$$

The difference in information levels of the seller and buyer is reflected in the inverse of depth (λ_3) as well. It is now decreasing in the level of noise in the information sold.

$$\lambda_3 = \frac{\sigma_v \kappa}{6\sigma_u}; \kappa = \sqrt[2]{8 - \frac{\sigma_\varepsilon^2}{\sigma_v^2}}$$

where $\kappa < 3 \Rightarrow \lambda_3 < \lambda_1$, though to satisfy the second order condition of $\lambda_3 > 0$ we need to have a positive κ, i.e., we consider only the positive root of $8 - \frac{\sigma_\varepsilon^2}{\sigma_v^2}$ where $\sigma_\varepsilon^2 < 8\sigma_v^2$. The addition of a noisily informed trader increases the market liquidity and prices are now less sensitive to trade than in the case with the single informed trader.

The profits in this case are given by:

$$E(\pi_S^3) = \frac{1}{\lambda_3}\left(\frac{(v-\mu_v)}{9}^2 + \frac{\sigma_\varepsilon^2}{36}\right); E(\pi_B^3) = \frac{1}{\lambda_3}\left(\frac{s-\mu_v}{3}\right)^2$$

$$E(\pi_S^3) + E\left(\pi_B^3\right) = \frac{\sigma_u}{\sigma_v \kappa}\frac{4(v-\mu_v)}{3}^2 + \frac{5\sigma_\varepsilon^2}{6}\frac{\sigma_u}{\sigma_v \kappa}$$

The next proposition shows that though the seller will never sell his information fully; he might be willing to sell a diluted version it. The result follows from the intuition that by selling exactly his true valuation the seller goes from being a monopolist to a being a Cournot competitor, which means lower total profits. But total profits can be higher if he maintains his informational advantage over the buyer as well as the market maker by selling a noisy signal.

Proposition 1 *The informed trader will never sell his information fully but he might sell a noisy signal of the true valuation.*

Proof. *From Admati and and Pfleiderer (1988), we know that*

$E\left(\pi_S^2\right) + E(\pi_B^2) < \pi^1 \Rightarrow$ *there will be no sale of information if the seller sells his signal without adding noise*

But if the seller sells a noisy signal; the sellers expected profits before trade will be greater than in the monopoly case if:

$E_s\left(\pi_S^3\right) + E_B\left(\pi_B^3\right) > \pi^1 \Leftrightarrow \frac{5\sigma_\varepsilon^2}{6} > (v-\mu_v)^2\left(\frac{\kappa}{2} - \frac{4}{3}\right)$

For large values of σ_ε^2 (i.e $\sigma_\varepsilon^2 \to 8\sigma_v^2$); we have the RHS of the constraint positive but the LHS will be negative since $\kappa \to 0$.

So for large enough values of σ_ε^2; the seller will be willing to sell noisy information. ■

Sale of noisy information is possible because, though the addition of another trader might lower the trading profits for the seller, they do not fall as low as in the symmetric information case.

4 Two Period Problem

We first present the buyer's belief updation rule in the following lemma. The lemma uses the normality of random variables and the Projection Theorem for Normal Variables. The buyer will update to a linear combination of his prior (s_1) and his new signal of the seller's trade $(x_{S1}^* + u_1)$.

Lemma 2 *In a linear equilibrium* $s_2 = s_1 + \gamma(x_{s1} + u_1 - \delta * x_{B1})$ *where* γ *and* δ *will be determined in the equilibrium.*

Proof. *We know that* $s_2 = E(v|s, x^*_{S1}+u_1)$ *and applying the projection thereom to this we get:*

$$\begin{aligned} s_2 &= E(v|s) + \frac{Cov(v, x^*_{S1} + u_1|s)}{var(x_{S1} + u_1|s)}(x^*_{S1} + u_1 - E_{B1}(x_{S1}|s)) \\ &= s_1 + \gamma(x^*_{S1} + u_1 - (\delta x^*_{B1})) \end{aligned}$$

Where $\frac{Cov(v,x^*_{S1}+u_1|s)}{var(x^*_{S1}+u_1|s)} = \gamma$ *and we get* $E_{B1}(x^*_{S1}|s) = \delta x^*_{B1}$ *assuming the seller's trade is linear and of the form* $\beta_{S1}(v - \mu_v) + \alpha_{S1}(s_1 - \mu_v)$ *and the buyers trade is also linear and of the form* $\beta_{B1}(v - \mu_v)$.

The exact form of δ *is given by:*

$$\begin{aligned} E_{B1}(x^*_{S1}) &= E_{B1}(\beta_{S1}(v - \mu_v) + \alpha_{S1}(s_1 - \mu_v) + u_1) \\ &= (\beta_{S1} + \alpha_{s1})(s_1 - \mu_v) \\ or\ \delta &= \frac{(\beta_{S1} + \alpha_{S1})}{\beta_{B1}} \end{aligned}$$

■

The buyer, having bought the signal, not only has more information than the market maker, he also learns more from the trade of the seller. Before trading in period 1, he knows only the signal. At the end of the first round of trading he observes a noisy signal of the sellers trade, $x^*_{S1} + u_1$, where u_1 is the level of noise trading. (The market maker also uses the total trade to update his belief - but he observes the noisy signal $x^*_{S1} + x^*_{B1} + u_1$).The proof uses the fact that the buyer (before trading) expects the seller to trade an amount proportional to his own trade, i.e., he expects the seller to trade (δx_{B1}), where δ is a function of the equilibrium intensity levels of the seller.

The next proposition characterises the two period equilibrium

Proposition 3 *A sequentially rational linear Bayesian Nash equilbrium of the game is of the form:*

$$\begin{aligned} p_i &= p_{i-1} + \lambda_i(x^*_{Si} + x^*_{Bi} + u_i) \\ x^*_{Si} &= \beta_{Si}(v - p_{i-1}) + \alpha_{Si}(s_i - p_{i-1}) \\ x^*_{Bi} &= \beta_{Bi}(s_i - p_{i-1}) \end{aligned}$$

where p_i *is the price in period* i *,(with* $p_0 = \mu_v$*)* *,*x^*_{Si} *is the equilibrium trade of the seller in period* i *and* x^*_{Bi} *is the equilibrium trade of the buyer in period* i*. It is characterised by the following system of equations:*

$\beta_{S2} = \frac{1}{2\lambda_2}$ $\qquad \alpha_{S2} = -\frac{1}{6\lambda_2}$

$\beta_{B2} = \frac{1}{3\lambda_2}$

$$\beta_{S1}=\frac{a}{d}\qquad \alpha_{S1}=\left(\frac{b}{d}+\frac{c}{d}\left(g-\frac{cf}{g}\right)^{-1}\left(e+\frac{(a+b)f}{d}\right)\right)$$
$$\beta_{B1}=\left(g-\frac{cf}{g}\right)^{-1}\left(e+\frac{(a+b)f}{d}\right)$$
where
$$a=1-\frac{\gamma}{6\lambda_2}-\frac{\lambda_1}{3\lambda_2}$$
$$b=\left(\frac{\gamma}{6\lambda_2}+\frac{\lambda_1}{3\lambda_2}\right)\frac{1}{3}$$
$$c=-\lambda_1-\frac{2}{\lambda_2}\left(\frac{\gamma}{6}+\frac{\lambda_1}{3}\right)\left(\frac{\gamma*\delta}{6}-\frac{\lambda_1}{3}\right)$$
$$d=2\lambda_1-\frac{2}{\lambda_2}\left(\frac{\gamma}{6}+\frac{\lambda_1}{3}\right)^2$$
$$e=1-\frac{2\lambda_1}{9\lambda_2}$$
$$f=\frac{2\lambda_1^2}{9\lambda_2}-\lambda_1$$
$$g=2\lambda_1-\frac{2\lambda_1^2}{9\lambda_2}$$
$$\delta=1-\frac{\gamma}{9\lambda_2-2\lambda_1}$$
$$\gamma==\frac{(\beta_{S1})var(v|s)}{\beta_{S1}var(v|s)+\sigma_u^2}$$
$$\lambda_1=\frac{(\beta_{S1}+(\alpha_{S1}+\beta_{B1})(1-q))\sigma_v^2}{(\beta_{S1}+(\alpha_{S1}+\beta_{B1})(1-q))^2\sigma_v^2+(\alpha_{S1}+\beta_{B1})^2(1-q)^2\sigma_\varepsilon^2+\sigma_u^2}$$
$$var(v|tr1)=\frac{((\alpha_{S1}+\beta_{B1})^2(1-q)^2\sigma_\varepsilon^2+\sigma_u^2)\sigma_v^2}{((\beta_{S1}+(\alpha_{S1}+\beta_{B1})(1-q))^2\sigma_v^2+(\alpha_{S1}+\beta_{B1})^2(1-q)^2\sigma_\varepsilon^2+\sigma_u^2)}$$
$$\lambda_2=\frac{cov(v,tr2|tr1)}{var(tr2|tr1)}$$
$$cov(v,tr2|tr1)=(\beta_{S2}+(\alpha_{S2}+\beta_{B2})\left(\gamma\ \beta_{S1}+(1-q)(1+\gamma\ \alpha_{S1}-\gamma\ \delta\ \beta_{B1})\right)var(v|tr1)$$
$$\begin{aligned}vat(tr2|tr1)=&(\beta_{S2}+(\alpha_{S2}+\beta_{B2})\left(\gamma\ \beta_{S1}+(1-q)(1+\gamma\ \alpha_{S1}-\gamma\ \delta\ \beta_{B1})\right)^2var(v|tr1)\\&+((\alpha_{S2}+\beta_{B2})\gamma+1)\sigma_u^2\\&+(\alpha_{S2}+\beta_{B2})^2(1-q)^2(1+\gamma\ \alpha_{S1}-\gamma\ \delta\ \beta_{B1})^2\sigma_\varepsilon^2\end{aligned}$$
if the second order conditions $\lambda_1\geqslant 0,\lambda_2\geqslant 0,g\geqslant 0,d\geqslant 0$ *are satisfied.*

The detailed proof for this proposition is in the appendix. The proof follows the backward induction argument. We first solve the traders' problem given the pricing rule and then derive the optimal pricing rule given the traders' strategies. The trader's second period problem is solved given the buyer's belief s_2 . The buyer's belief, as we saw in the lemma, depends on the signal he bought and the seller's first period trade. The seller knows s_2 in equilibrium. So given any values for p_1 and s_2 (both values are known perfectly by the seller and buyer), the second period results are as given in the proposition. Given the second period result, we find the equilibrium for the period one problem. Here we use the conjectured belief updating rule from the lemma to solve the problem for the seller. The seller has to know exactly the impact he will have on the buyer's beliefs. Using the signal updation and also the pricing rule rule as given, we solve the period one problem.

Once the traders' optimal strategies are known, we can solve for the $\lambda_i's$ and γ using the projection theorem. The inequality constraints ensure that the solution is a maximum. They come from the traders' second order conditions in the two periods.

5 Positive (Negative) Trade with Negative (Positive) Information

By positive information I mean $v - \mu_v > 0$ and similarly negative information means $v - \mu_v < 0$. In the Kyle (1985) model the trader would buy if and only if he had positive information and similarly sell iff he had negative information. This is so because in that model the trade is linear only in his own information; and also because the trading intensity parameter,β_{S1}, is positive as an implication of the second order condition. In short, only if the trader knows the liquidation value of the asset is higher than the market maker's ex-ante prior in that period, would he buy, otherwise he would sell. This result also holds in Foster and Viswanathan (1996), where many traders receive i.i.d. signals and they observe only their own signal realisation. Since they do not observe any other signal besides their own signal, and they know signals are i.i.d., they expect all other strategic traders to place net buy order's similar to theirs. In their model, the priors of the traders are also changing each period but as long as the traders expected valuation is higher than the market maker's prior, the trader buys and otherwise he sells.

In the current model there are two pieces of information (and they are not i.i.d) : the seller's knowledge of the true valuation and the signal that he sells to the buyer. The seller has the added advantage of knowing both pieces of information. The buyer knows only the value of the signal he receives and he trades accordingly. If his information (the signal he buys) is above the market maker'e ex-ante expectation, he buys and otherwise he sells. But the trading rule of the seller of information is not so straightforward anymore. His trade is linear in both the true liquidation value of the good and the signal that he sells. In equilibrium, the seller would know exactly how much the buyer would trade and he would, therefore, know exactly what the price impact of that trade would be. He will take that into account when making his trading decision and the direction of his trade is no longer determined solely by his valuation. The result is presented in the following proposition

Proposition 4 *The seller of information might sell (buy) when $v - \mu_v > 0$ $(v - \mu_v < 0)$.*
Proof. *The result can be seen from the following:*

$$\begin{aligned}
\Pr[x_{S1} &> 0 | v - \mu_v < 0] = \Pr[(\beta_{S1} + \alpha_{S1}(1-q))(v - \mu_v) + \alpha_{S1}(1-q)\varepsilon > 0] \\
&= \Pr[\varepsilon > (\beta_{S1} + \alpha_{S1}(1-q))(\mu_v - v)(\alpha_{S1}(1-q))^{-1}] \\
&> 0
\end{aligned}$$

Similarly the result would also hold for $x_{S1} < 0$ given $v - \mu_v > 0$. ■

The fact that the probability of trading in a direction opposite to information is strictly positive is of course, a result of the normality assumptions. What the proposition implies, is that, in this market with sale of information, we could observe both strategic traders buying the stock even when the more informed trader knows that the stock is overpriced. The buyer's direction of trade influences the direction of trade of the seller substantially. If the realisation of the signal sold by the seller was large enough and in the opposite direction as compared to the true vaule, it could swamp the effect of the true valuation in the seller's trade. This could lead to patterns of trade very different from the ones studied so far.

6 Information "Hiding" by the Seller

At the end of period one, the seller's trade is used by both the buyer and the market maker to update their beliefs. To see this clearly, note that the total trade in period one is given by:

$$trade1 = \beta_{S1}(v - \mu_v) + (\alpha_{S1} + \beta_{B1})(s_1 - \mu_v) + u_1$$

Given this and the fact that the buyer already knows the value of $\beta_{S1}, \alpha_{S1}, \beta_{B1}, s_1$ and μ_v; the buyer can extract a signal that we call y given by:

$$y = \beta_{S1} v + u_1$$

Equivalently he knows the unbiased estimator of v called y' which is given by:

$$y' = v + \frac{u_1}{\beta_{S1}}$$

Since we know $u_1 \sim N(0, \sigma_u^2)$ and $v \sim N(\mu_v, \sigma_v^2)$ and that they are independent, we get that y' is also normally distributed with

$$y' \sim N(\mu_v, \sigma_v^2 + \frac{\sigma_u^2}{\beta_{S1}}) \quad \text{and} \quad y'|s \sim N(s_1, var(v|s) + \frac{\sigma_u^2}{\beta_{S1}})$$

Clearly, the smaller is the value of β_{S1}, the larger is the variance of $y'|s$. In other words, the seller might have the incentive to trade less intensively on his information to make the buyer's signal noisier. The signal will always be an unbiased estimator of v as long as the seller plays his equilibrium strategy but it can be made noisier by a smaller β_{S1}. In other words, the seller might have the incentive to "hide" his information so as to not give out information to the buyer. How intensively the buyer uses y will of course depend on γ (where γ measures how sensitive the buyer's signal updation at the end of trading period 1 is to the seller's trade). The higher is γ, the more is the impact of the seller's trade on his signal for period two.

The market maker also gets a noisy signal of v from the total trade, which we call z here. z is given by:

$$\begin{aligned} z &= (\beta_{S1} + (\alpha_{S1} + \beta_{B1})(1-q))(v) + (\alpha_{S1} + \beta_{B1})(1-q)\varepsilon + u_1 \\ z' &= v + ((\alpha_{S1} + \beta_{B1})(1-q)\varepsilon + u_1)(\beta_{S1} + (\alpha_{S1} + \beta_{B1})(1-q))^{-1} \end{aligned}$$

Again, the smaller is the value of β_{S1}, the larger is the variance of z'. But in this case the effect is of a smaller magnitude. z' is in any case a noisier estimate of v than y'. The buyer, having more prior information than the market maker, is able to extract more information from the trade than the latter. The market maker's sensitvity to trade is given by λ_1. But if markets are very liquid and $\lambda_1 \approx 0$ then the seller would not care so much about the learning from the buyer, because with $\lambda_1 \approx 0$ they can both make huge profits. In a sense, as λ_1 approaches 0, the competition amongst the two informed traders becomes negligible. The seller worries about divulging too much information to the buyer only if the information learned by the buyer would get reflected in the prices which would lead to lower profits for him.

The seller is defined to be myopic if he assumes $\gamma = 0$. That is, he does not take into account the impact of his trade on the learning by the buyer, though he does know the impact of his trade in prices. Let β_{S1}^{myopic} be the myopic seller's trading intensity in the first period, then we have:

Proposition 5 *$\beta_{S1} < \beta_{S1}^{myopic}$ iff $\frac{\gamma}{\lambda_1} < 2\left(1+\frac{\lambda_1}{3\lambda_2}\right)\left(1-\frac{\lambda_1}{3\lambda_2}\right)^{-1}$ given γ, λ_1 and λ_2. (If $g(\lambda_1,\lambda_2) = \left(1+\frac{\lambda_1}{3\lambda_2}\right)\left(1-\frac{\lambda_1}{3\lambda_2}\right)^{-1}$, then $\frac{dg(\lambda_1,\lambda_2)}{d\lambda_1} > 0$.)*

Proof. *The proof uses the fact that:*

$$\begin{aligned}
\beta_{S1} &= \left(1-\frac{\gamma}{6\lambda_2}-\frac{\lambda_1}{3\lambda_2}\right)\left(2\lambda_1-\frac{2}{\lambda_2}\left(\frac{\gamma}{6}+\frac{\lambda_1}{3}\right)^2\right)^{-1} \\
\beta_{S1}^{myopic} &= \left(1-\frac{\lambda_1}{3\lambda_2}\right)\left(2\lambda_1-\frac{2}{\lambda_2}\left(\frac{\lambda_1}{3}\right)^2\right)^{-1}
\end{aligned}$$

Where β_{S1} is calculated by solving the problem with the constraint $\gamma = 0$. ■

The condition in the proposition shows that the presence of updating by the buyer will reduce the trading intensity of the seller if λ_1 is high enough. This is so because the high λ_1 reduces the LHS of the condition and increases the RHS, implying there would be a cut off $\overline{\lambda_1}$(which would depend on λ_2 and γ) such that for $\lambda_1 \geqq \overline{\lambda_1}$, the seller of information would want to trade less intensively on his information as compared to the benchmark case where he does not take the buyer's learning into account. On the other hand with lower levels of λ_1, there would be lower competition between the two players (as their trades have a low impact on prices), and so if λ_1 is low enough the seller might be willing to trade more aggresively than in the benchmark case.

The proof can be seen illustrated in Figure 1 where the line on top is the curve for β_{S1}^{myopic} and the one below is β_{S1}. On the x-axis is σ_u^2 varying from 1 to 10 and all other variances are fixed at 1. As we can see, $\beta_{S1} < \beta_{S1}^{myopic}$ for all these values of σ_u^2. Increasing the variance of noise trading would lead to lower levels for λ_1, λ_2 and γ.

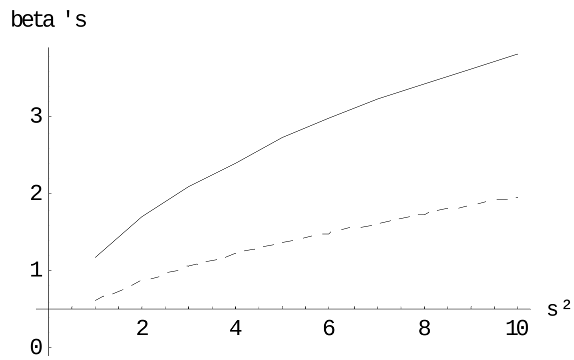


Figure 1: - - - beta — beta myopic

7 Efficiency of Prices

Prices are more efficient if they reveal more information and a measure of efficiency is the variance of the value of the asset given the prices. This is the variance of the information that the market maker has. The less variable his information the greater is the probability of prices being closer to the true liquidation value. The more information prices incorporate the less should be this variance.

A favourable impact of information sale is that prices are more efficient as is seen in the following result.

Proposition 6 *Prices are more efficient in a model with sale of noisy information than with a single trader in a one shot game.*

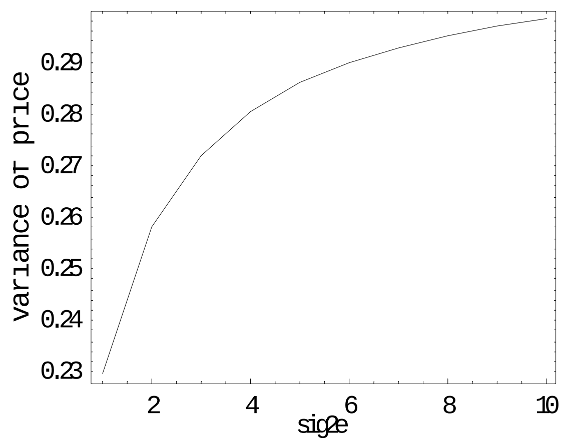
Proof. *In a one period one monopolistic player model* $var(v|p^1) = \frac{\sigma_v^2}{2}$.

In a one period one sale of information model $var(v|p^3) = \frac{\sigma_v^2}{3}$ ■

The increased information in the trades is reflected in the prices as well. The final prices will be closer to the liquidation value. With sale of information the sum of profits for the informed traders is higher even though prices are more efficient. This is bad news in some ways for those who trade for non-informational reasons, the traders modeled as "noise" traders in our model. The higher expected profits of the traders might mean lower profits for the noise traders.

The efficiency result is interesting for two reasons. One is that even though in the trading pattern we observe "hiding" by the seller and the possibility of opposite trades is positive, we still find prices converging to the truth faster. Just the fact that there is another trader with some information makes the price system a lot more efficient even though the seller might now be revealing a lot less of his true information. The second important thing to note is that, at least in the one period game, the variance of prices is independent of the variance of the signal.

In longer time horizons the variance of prices would need to be evaluated numerically. In figure 2 we show the results for the 2 period game for increasing levels of noise of the signal. The variance of second period prices is increasing in the variance of the the signal. All other variances are fixed at 1.



8 Conlusion

The paper shows the possibility of the sale of direct information even in the case of risk-nuetral investors. It also shows that the seller might trade less aggressively on his information. But prices will be more efficient.

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A Appendix Proof of Propostion

A.1 Traders Problem

The traders wish to maximize the end of two periods of profits given the market makers linear pricing rule.

I will solve the traders problem backwards.

A.1.1 Period 2 problem (last period)

Trader 1's maximization problem:

$$\underset{x_{s2}}{Max}\ \pi_{12} = E\left((v - p_2)\, x_{s2}\right)$$

or

$$\underset{x_{s2}}{Max} \pi_{12} = E\left((v - p_1 - \lambda_2(x_{s2} + x_{B2} + u_2))\, x_{s2}\right)$$

which yeilds the first order condition (after taking expectations)

$$2\lambda_2 x_{s2} = v - p_1 - \lambda_2 x_{B2} \tag{1}$$

Similarly the other players problem is:

$$\underset{x_{B2}}{Max} \pi_{22} = E\left((v - p_1 - \lambda_2(x_{s2} + x_{B2} + u_2))\, x_{B2}\right)$$

which yeilds the first order condition:

$$2\lambda_2 x_{B2} = s_2 - p_1 - \lambda_2 E(x_{s2}) \tag{2}$$

Solving for $x_{s1}$and x_{B1} from equation (1) and (2) we get:

$$\begin{aligned} x_{s2} &= \frac{v - p_1}{2\lambda_2} - \frac{s_2 - p_1}{6\lambda_2} \\ x_{B2} &= \frac{s_2 - p_1}{3\lambda_2} \end{aligned}$$

and the expected profit is:

$$\begin{aligned} \pi_{12} &= \frac{1}{\lambda_2}\left(\frac{v - p_1}{2} - \frac{s_2 - p_1}{6}\right)^2 \\ \pi_{22} &= \frac{1}{\lambda_2}\left(\frac{s_2 - p_1}{3}\right)^2 \end{aligned}$$

The second order condition is simply $\lambda_2 \geqq 0$.

A.1.2 Signal Updation in Period 2

The updated signal s_2 is the second traders expectation of the final value v given his information till the beginning of period one which comprises of his signal s and the trade in period 1 by the other player which is $x_{s1} + u_1$.The second term is distribued as: $x_{s1} + u_1 \sim N(E_{21}(x_{s1}), \sigma_u^2)$. Since all the variables are normally distributed we can apply the projection theorem for normal variables

$$
\begin{aligned}
s_2 &= E(v|s, x_{s1} + u_1) \\
&= E(v|s) + \frac{Cov(v, x_{s1} + u_1|s)}{var(x_{s1} + u_1|s)}(x_{s1} + u_1 - E_{21}(x_{s1})) \\
&= s_1 + \gamma(x_{s1} + u_1 - (\delta * x_{B1}))
\end{aligned}
$$

The last line comes from simply setting $\frac{Cov(v,x_{s1}+u_1|s)}{var(x_{s1}+u_1|s)} = \gamma$ and also assuming that the second trader expects the first players traders trade to be a fraction δ of his trade x_{21}.Solving for ce gives:

$$
\begin{aligned}
\gamma &= \frac{Cov(v, x_{s1} + u_1|s)}{var(x_{s1} + u_1|s)} \\
&= \frac{Cov(v, \beta_{s1}(v - \mu_v) + \alpha_{s1}(s_1 - \mu_v) + u_1|s)}{var(\beta_{s1}(v - \mu_v) + \alpha_{s1}(s_1 - \mu_v) + u_1|s)} \\
&= \frac{(\beta_{s1})var(v|s)}{\beta_{s1}^2 var(v|s) + \sigma_u^2}
\end{aligned}
$$

where

$$
\begin{aligned}
s_1 &= E(v|s) \\
&= q\mu_v + (1 - q)s \\
q &= \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2} \\
var(v|s) &= \frac{\sigma_\varepsilon^2 \sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}
\end{aligned}
$$

We need to also know the expression for δ which will depend on the period zero's trading strategies:

$$
\begin{aligned}
E_{21}(x_{s1}) &= \delta * x_{B1} \\
E_{20}(\beta_{s1}(v - \mu_v) + \alpha_{s1}(s_1 - \mu_v) + u_1) &= \delta * x_{B1} \\
(\beta_{s1} + \alpha_{s1})(s_1 - \mu_v) &= \delta * \beta_{B1}(s_1 - \mu_v) \\
\delta &= \frac{\beta_{s1} + \alpha_{s1}}{\beta_{B1}}
\end{aligned}
$$

A.1.3 Period 1 Problem (the first period)

The traders problem are:

$$
\begin{aligned}
\underset{x_{s1}}{Max}\ \pi_{11} &= E\left(\left(v-p_1\right)x_{s1}+\pi_{12}\right)\\
\underset{x_{B1}}{Max}\ \pi_{21} &= E\left(\left(v-p_1\right)x_{B1}+\pi_{22}\right)
\end{aligned}
$$

which after replacing for π_{11} and π_{11} are:

$$
\begin{aligned}
\underset{x_{s1}}{Max}\ \pi_{11} &= E\left(\left(v-p_1\right)x_{s1}+\frac{1}{\lambda_2}\left(\frac{v-p_1}{2}-\frac{s_2-p_1}{6}\right)^2\right)\\
\underset{x_{B1}}{Max}\ \pi_{21} &= E\left(\left(v-p_1\right)x_{B1}+\frac{1}{\lambda_2}\left(\frac{s_2-p_1}{3}\right)^2\right)
\end{aligned}
$$

now replacing for $p_1$and s_1 we get:

$$
\begin{aligned}
\underset{x_{s1}}{Max}\ \pi_{11} &= E\begin{pmatrix} \left(v-\mu_v-\lambda_1(x_{s1}+x_{B1}+u_1)\right)x_{s1} \\ +\frac{1}{\lambda_2}\left(\frac{v-p_1}{2}-\frac{s_1+\gamma(x_{s1}+u_1-(\delta*x_{B1}))-p_1}{6}\right)^2 \end{pmatrix}\\
\underset{x_{B1}}{Max}\ \pi_{21} &= E\begin{pmatrix} \left(v-\mu_v-\lambda_1(x_{s1}+x_{B1}+u_1)\right)x_{B1} \\ +\frac{1}{\lambda_2}\left(\frac{s_1+\gamma(x_{s1}+u_1-(\delta*x_{B1}))-p_1}{3}\right)^2 \end{pmatrix}
\end{aligned}
$$

and now the first order conditions are of the form:

$$
\begin{aligned}
a(v-\mu_v)+b(s_1-\mu_v)+cx_{B1} &= dx_{s1}\\
e(s_1-\mu_v)+f(E_{21}(x_{s1})) &= gx_{B1}
\end{aligned}
$$

where:

$$
\begin{aligned}
a &= 1 - \frac{\gamma}{6\lambda_2} - \frac{\lambda_1}{3\lambda_2} \\
b &= \left(\frac{\gamma}{6\lambda_2} + \frac{\lambda_1}{3\lambda_2}\right)\frac{1}{3} \\
c &= -\lambda_1 - \frac{2}{\lambda_2}\left(\frac{\gamma}{6} + \frac{\lambda_1}{3}\right)\left(\frac{\gamma * \delta}{6} - \frac{\lambda_1}{3}\right) \\
d &= 2\lambda_1 - \frac{2}{\lambda_2}\left(\frac{\gamma}{6} + \frac{\lambda_1}{3}\right)^2 \\
e &= 1 + \frac{2}{3\lambda_2}\left(-\frac{\gamma * \delta}{3} + \frac{\lambda_1}{3}\right) \\
f &= \frac{2}{3\lambda_2}\left(-\frac{\gamma * \delta}{3} + \frac{\lambda_1}{3}\right)(\gamma + \lambda_1) - \lambda_1 \\
g &= 2\lambda_1 - \frac{2}{9\lambda_2}(\gamma * \delta + \lambda_1)^2
\end{aligned}
$$

The solution will be:

$$
\begin{aligned}
x_{s1} &= \frac{a}{d}(v - \mu_v) + \left(\frac{b}{d} + \frac{c}{d}\left(g - \frac{cf}{d}\right)^{-1}\left(e + \frac{(a+b)\,f}{d}\right)\right)(s_1 - \mu_v) \\
x_{B1} &= \left(g - \frac{cf}{d}\right)^{-1}\left(e + \frac{(a+b)\,f}{d}\right)(s_1 - \mu_v) \\
E_{21}(x_{s1}) &= \left(g - \frac{cf}{d}\right)^{-1}\left(\frac{c * e}{d} + \frac{(a+b)\,g}{d}\right)(s_1 - \mu_v)
\end{aligned}
$$

The second order condition are $g \geqslant 0$ and $d \geqslant 0$ which lead to the results $\lambda_1 \geqq 0$ and $9\lambda_2 \geqq \lambda_1$

Profits for the first trader:

$$
\begin{aligned}
x_{s1} &= \beta_{s1}(v-\mu_v)+\alpha_{s1}(s_1-\mu_v) \\
x_{B1} &= \beta_{B1}(s_1-\mu_v) \\
tr1 &= \beta_{s1}(v-\mu_v)+(\alpha_{s1}+\beta_{B1})(s_1-\mu_v)+u_1 \\
p_1 &= \mu_v+\lambda_1\beta_{s1}(v-\mu_v)+\lambda_1(\alpha_{s1}+\beta_{B1})(s_1-\mu_v)+\lambda_1 u_1 \\
s_2 &= s_1+\gamma(\beta_{s1}(v-\mu_v)+u_1-\beta_{s1}(s_1-\mu_v)) \\
\pi_{12} &= \frac{1}{\lambda_2}\left(\frac{v-p_1}{2}-\frac{s_2-p_1}{6}\right)^2 \\
&= \frac{1}{\lambda_2}\left(\begin{array}{c} \frac{(1-\lambda_1\beta_{s1})(v-\mu_v)-\lambda_1(\alpha_{s1}+\beta_{B1})(s_1-\mu_v)-\lambda_1 u_1}{2} \\ -\frac{(\gamma\beta_{s1}-\lambda_1\beta_{s1})(v-\mu_v)+(1-\lambda_1(\alpha_{s1}+\beta_{B1})-\gamma\beta_{s1})(s_1-\mu_v)+\gamma u_1-\lambda_1 u_1}{6}\end{array}\right)^2 \\
&= \frac{1}{\lambda_2}\left(\begin{array}{c} \left(\frac{1}{3}-\frac{\lambda_1\beta_{s1}}{3}-\frac{\gamma\beta_{s1}}{6}\right)(v-\mu_v)+\left(\frac{\gamma\beta_{s1}}{6}-\frac{\lambda_1(\alpha_{s1}+\beta_{B1})}{3}-\frac{1}{6}\right)(s_1-\mu_v) \\ -\frac{\lambda_1 u_1}{3}-\frac{\gamma u_1}{6}\end{array}\right)^2 \\
(v-p_1)\,x_{s1} &= (v-\mu_v)^2(1-\lambda_1\beta_{s1})\beta_{s1}-(s_1-\mu_v)^2\lambda_1\alpha_{s1}(\alpha_{s1}+\beta_{B1}) \\
&\quad +(v-\mu_v)(s_1-\mu_v)\left[(1-\lambda_1\beta_{s1})\alpha_{s1}-\lambda_1\beta_{s1}(\alpha_{s1}+\beta_{B1})\right] \\
\pi_{11} &= (v-p_1)\,x_{s1}+\pi_{12} \\
&= (v-\mu_v)^2\left[(1-\lambda_1\beta_{s1})\beta_{s1}+\left(\frac{1}{3}-\frac{\lambda_1\beta_{s1}}{3}-\frac{\gamma\beta_{s1}}{6}\right)^2\right] \\
&\quad +(s_1-\mu_v)^2\left[\left(\frac{\gamma\beta_{s1}}{6}-\frac{\lambda_1(\alpha_{s1}+\beta_{B1})}{3}-\frac{1}{6}\right)^2-(\lambda_1\alpha_{s1}+\lambda_1\alpha_{s1}\beta_{B1})\right] \\
&\quad +(v-\mu_v)(s_1-\mu_v)\left[\begin{array}{c} 2\left(\frac{1}{3}-\frac{\lambda_1\beta_{s1}}{3}-\frac{\gamma\beta_{s1}}{6}\right)\left(\frac{\gamma\beta_{s1}}{6}-\frac{\lambda_1(\alpha_{s1}+\beta_{B1})}{3}-\frac{1}{6}\right) \\ +\alpha_{s1}-2\lambda_1\beta_{s1}\alpha_{s1}-2\lambda_1\beta_{s1}\beta_{B1}\end{array}\right]
\end{aligned}
$$

A.2 Market Makers Problem

The market maker just sets prices equal to the expected value of the liquidation value given his information which consists of the history of order flows.

A.2.1 Prices in period 1:

$$p_1 = E(v|x_{s1}+x_{B1}+u_1)$$

where total trade is:

$$
\begin{aligned}
tr1 &= x_{s1}+x_{B1}+u_1 \\
&= \beta_{s1}(v-\mu_v)+(\alpha_{s1}+\beta_{B1})(s_1-\mu_v)+u_1 \\
&= (\beta_{s1}+(\alpha_{s1}+\beta_{B1})(1-q))(v-\mu_v)+(\alpha_{s1}+\beta_{B1})(1-q)\varepsilon+u_1
\end{aligned}
$$

Since total trade is a function of $v, \varepsilon$and u_1 which are all normally distributed we can apply the projection theorem.

$$p_1 = \mu_v + \lambda_1 (x_{s1} + x_{B1} + u_1)$$

where

$$\begin{aligned}
\lambda_1 &= \frac{cov(v, tr1)}{var(tr1)} \\
cov(v, tr1) &= (\beta_{s1} + (\alpha_{s1} + \beta_{B1})(1-q))\sigma_v^2 \\
var(tr1) &= (\beta_{s1} + (\alpha_{s1} + \beta_{B1})(1-q))^2\sigma_v^2 \\
&\quad +(\alpha_{s1} + \beta_{B1})^2(1-q)^2\sigma_\varepsilon^2 + \sigma_u^2 \\
var(v|tr1) &= \frac{((\alpha_{s1} + \beta_{B1})^2(1-q)^2\sigma_\varepsilon^2 + \sigma_u^2)\sigma_v^2}{\begin{pmatrix} (\beta_{s1} + (\alpha_{s1} + \beta_{B1})(1-q))^2\sigma_v^2 \\ +(\alpha_{s1} + \beta_{B1})^2(1-q)^2\sigma_\varepsilon^2 + \sigma_u^2 \end{pmatrix}}
\end{aligned}$$

A.2.2 Prices in Period 2:

Doing the same as before, the total trade in this period is:

$$\begin{aligned}
tr2 &= x_{s2} + x_{B2} + u_2 \\
&= \beta_{s2}(v - p_1) + (\alpha_{s2} + \beta_{B2})(s_2 - p_1) + u_2 \\
&= (\beta_{s2} + (\alpha_{s2} + \beta_{B2})(\gamma * \beta_{s1} + (1-q)(1 + \gamma * \alpha_{s1} - \gamma * \delta * \beta_{B1})))v \\
&\quad +(\alpha_{s2} + \beta_{B2})\gamma * u_1 + u_0 \\
&\quad +(\alpha_{s2} + \beta_{B2})(1-q)(1 + \gamma * \alpha_{s1} - \gamma * \delta * \beta_{B1})\varepsilon \\
&\quad -((\alpha_{s2} + \beta_{B2})(\gamma * \beta_{B1} - q + (1-q)(\gamma * \alpha_{s1} - \gamma * \delta * \beta_{B1})))\mu_v \\
&\quad -p_1(\beta_{s2} + (\alpha_{s2} + \beta_{B2}))
\end{aligned}$$

And now solving for λ_2 :

$$\begin{aligned}
\lambda_2 &= \frac{cov(v, x_{s2} + x_{B2} + u_2 | x_{s1} + x_{B1} + u_1)}{var(x_{s2} + x_{B2} + u_2 | x_{s1} + x_{B1} + u_1)} \\
cov(tr2|tr1) &= \left(\begin{array}{c} \beta_{s2} + (\alpha_{s2} + \beta_{B2}) \\ \left(\begin{array}{c} \gamma * \beta_{s1} \\ +(1-q)(1 + \gamma * \alpha_{s1} - \gamma * \delta * \beta_{B1} \end{array} \right) \end{array} \right) var(v|tr1) \\
var(tr2|tr1) &= \left(\begin{array}{c} \beta_{s2} + (\alpha_{s2} + \beta_{B2}) \\ \left(\begin{array}{c} \gamma * \beta_{s1} \\ +(1-q)(1 + \gamma * \alpha_{s1} - \gamma * \delta * \beta_{B1} \end{array} \right) \end{array} \right)^2 var(v|tr1) \\
&\quad +((\alpha_{s2} + \beta_{B2})\gamma + 1)\sigma_u^2 \\
&\quad +(\alpha_{s2} + \beta_{B2})^2(1-q)^2(1 + \gamma * \alpha_{s1} - \gamma * \delta * \beta_{B1})^2\sigma_\varepsilon^2
\end{aligned}$$