FACTOR SAVING INNOVATIONS AND CAPITAL INCOME SHARE IN OLG MODELS

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Abstract

We present an endogenous growth model where innovations are factor-saving. We model the choice of technologies in an Overlapping Generations Model where any technology can be adopted paying a cost. Markets are competitive and marginal productivity of factors determines factor prices; therefore, innovations affect factor income shares. The main results are the following:

(i) The elasticity of output with respect to reproducible factors (physical and human capital) depends on the factor abundance of the economies. (ii) The income share of reproducible factors increases with the stage of development. (iii) Depending on the initial conditions, in some economies the production function converges to an AK, while in other economies long-run growth is zero. (iv) In some economies technological change may reduce future capital labor ratio and future income.

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1 Introduction.

Since the seminal Solow growth model (Solow, 1956), the behavior of the Solow residual has received a lot of attention in the economic literature. Endogenous growth theorists (Romer, 1986 and Lucas, 1988) assume that the Solow residual can be explained by the evolution of total factor productivity (TFP), which they model as a function of different variables (physical capital, human capital, etc.).

From a different perspective, technological improvements are explained as changes in factor intensity. Many papers on income distribution and labor economics suggest that an increase in the supply of skilled labor can generate skill-biased technological change (Kennedy, 1964 and Krugman 1997, among others). Indeed, many argue that in the last few decades, there has been *human capital-using* and *raw labor-saving* technological change (see Kiley, 1997; Krusell et.al, 2000 or Acemoglu, 2002). In the same line, research in economic history provides evidence that the Industrial Revolution was accompanied by *capital-using* and *labor-saving* technological change (Cain and Paterson, 1981). Moreover, in both cases innovations were preceded by a change in factor abundance.

In this paper we subscribe to the explanation of *capital-using* and *labor-saving* innovations and model the choice of new technologies in an Overlapping Generations Model. This setting allows us to analyze the distributive effects of *capital-using* and *labor-saving* innovations and see the interactions between income distribution and innovations.

The main results of the model are the following: (i) The elasticity of output with respect to reproducible factors (physical and human capital) depends on the factor abundance of the economies. (ii) The income share of reproducible factors increases with the stage of development. (iii) Depending on the initial conditions, in some economies the production function converges to AK, while in other economies long-run growth is zero. (iv) In some economies technological change may reduce future income.

Three pieces of empirical evidence motivates our work:

1. In the field of empirical economic growth, Durlauf and Johnson (1995) and Duffy and Papageorgiou (2000) find that as economies grow, their technologies become more intensive in reproducible factors, that is, the elasticity of output with respect to reproducible factors is higher in rich economies.

2. With regard to the behavior of factor income shares, we know that: (i) In developed countries the share of agriculture in total output is usually smaller than it is in developing countries. By the same token, the share of agriculture in total output is reduced as economies grow. Since land is a major input in agriculture but not in other sectors, these facts suggest that land income share may decrease with the stage of development. Consistently, in USA, from 1900 to 1945, the share of land in Net National Product was reduced from 59% to 29% (see Rhee, 1991). (ii) Over the past 60 years, the US relative supply of skilled work has increased rapidly. However, there has not been a downward trend for the returns to college education. On the contrary, over this period, the college premium has increased (see Krueger, 1999; Krusell et.al., 2000 and Acemoglu 2002). (iii) It has been argued that labor income share does not decrease or increase with development (Gollin, 2002). However, the standard measure of labor income share includes skilled and unskilled labor income share, that is, it includes human capital. In the same way, the standard measure of capital income share includes land income share. Therefore, given the behavior of unskilled labor and land income share, it seems that the income share of non-reproducible factors (land and unskilled labor) has decreased, while the income share of reproducible factors has increased during the 20th century.

3. Blanchard (1998) notes that since the early 80's a dramatic decrease in labor income share has occurred in Europe (5 to 10 percentage points of GDP) and suggests that this decline could be explained by non-neutral changes in technology.

To account for these facts we need a model of biased technological change where factor income shares are endogenous. We use a standard set-up (Cobb-Douglas production function) where factor prices are determined by the marginal productivity of factors. As a consequence, *labor-saving* innovations reduce labor income share and increase capital income share. In more general terms, the model predicts that the income share of non-reproducible factors decreases with the stage of development, while the income share of reproducible factors grows with the stage of development. To analyze the distributive effects of *labor-savings* and *capital-using* innovations, we differentiate between capital owners and workers by using an Overlapping Generations Model. In this setting innovations may reduce the income and savings of young people as well as the rate of economic growth.

The relation between income distribution and growth has been studied thoroughly (Persson and Tabellini, 1994; Galor and Tsidon 1997; Galor and Moav, 2000; and Hassler and Rodríguez, 2000, among others). The novelty of the paper at hand comes from the fact that here technological change affects the marginal productivity of factors and, as a result, factor income shares. Similarly, movements in factor income shares generate changes in savings and capital accumulation. Zeira (1998) and Acemoglu (2002) among others present models with this type of technological change but do not explain long-run growth through this relation. Boldrin and Levine (2002a) provide a model of perfect competition, where long-run growth is completely explained by factor saving innovations but they do not consider the effect of technology on capital income share and income distribution.

This paper is also related to Bertola (1993 and 1996). He finds that in a standard model with infinitely-lived agents there is a positive relation between capital income share and economic growth (Bertola, 1993), but that in an Overlapping Generations Model, a relationship of the opposite sign may appear (Bertola, 1996). We build on Bertola's argument by making endogenous the capital income share. Since we assume competitive markets, capital income share is determined by the capital intensity of the technology. Other models where capital intensity is endogenous are found in Jones and Manuelli (1992), Klump and De la Grandville (2000), and Zuleta (2004) but such endogeneity is independent of the producers' decisions. In our model, technical advances come from the rational decisions of people as capital owners can choose the capital intensity of the technology.

In our framework, agents live two periods. In the first period they work, consume and save. In the second period they use their savings to build capital and create or adopt technologies, produce, consume, leave a bequest to their children, and die. Finally, we assume that the capital owner has to take care of administration and supervision, so for each firm it is optimal to produce with only one plant and consequently only one technology is used at a time.

Technology is embodied in capital goods and capital goods of better quality are more costly. Capital abundant countries, where the capital intensity of the technology is higher, produce more output. As a result, capital abundant countries have more incentives to increase capital intensity.

The effect that *labor-saving* innovations have on wages may be positive or negative. A positive effect arises from an increase in output, while a negative effect arises from a reduction on labor income share. The net effect depends on the factor abundance of the economy. For rich economies a *labor-saving* innovation leads to an increase in wages, while in poor economies a *labor-saving* innovation leads to a reduction in wages. Similarly, the effect on savings and capital accumulation depends on the behavior of wages and bequests. Thus, if productivity and bequests are high enough, then in rich economies, as the capital labor ratio grows, technology becomes more capital intensive and as the economy becomes more capital abundant, the incentives for *labor-saving* and *capital-using* technological change grow stronger. In this way, in the long-run the production function may converge to an AK. On the other hand, labor abundant economies may be trapped in a steady state.

Although we consider a model with only two factors, the result on the evolution of the factor intensity can be extended to human capital (or any reproducible factor), that is, as economies become human capital abundant the technology becomes more human capital intensive.

The paper is organized in 5 sections. In the next section we present the basic OLG model and the results. In the third section we consider the effects of adopting cost-less capital intensive technologies. In the fourth section we show some simulations that illustrate the different paths that economies can follow depending on the initial conditions. Section five concludes.

2 The Model.

In this section we present an OLG model where agents can be altruists and leave bequests. For simplicity, we assume a logarithmic utility function, so current savings are completely determined by current levels of capital and technology.

Savings are used to build or buy capital goods and these goods can be of different qualities. Technology is embodied in capital goods, and technological changes are *labor-saving* and *capital-using*. In particular, we consider a Cobb-Douglas production function ($Y = AK^{\alpha}L^{1-\alpha}$) and assume that technologies are differentiated by their capital intensity α. Any technology has a positive cost which depends on the desired α. This cost is paid by the old people before the production process. Thus, the capital used in the production process is a share of their savings ($\delta(\alpha)$). That is, if an agent devotes s units of output to build a stock of capital of quality α, the amount of capital of quality α is equal to $\delta(\alpha)s$. We also assume the existence of a primitive technology (α_0), whose cost is equal to the price of the consumption good ($\delta(\alpha_0) = 1$).

The cost of a technology α can be interpreted in three different ways: (i) The cost of inventing and implementing a technology. (ii) The cost of copying a new technology and building a similar capital good. (iii) The price premium that has to be paid in order to acquire a higher quality capital good. In a market economy the costs described in (ii) and (iii) are likely to be the same. If we assume that technology is not rival, then the costs described in (ii) and (iii) are likely to be smaller than the one described in (i). However, if we assume that technology is embodied in goods and that it is costly to reverse engineer (disembody) and appropriate, the difference between (i) and (ii) is substantially reduced (see Boldrin and Levine, 2002b). In any case, since we want to derive conclusions for developing countries, which are unlikely to be the source of new inventions, we can ignore interpretation (i) and assume that capital goods of different qualities are available in the market. Thus, $\frac{1}{\delta(\alpha)}$ is the price of a capital good of quality α.

The share of savings devoted to produce the final good ($\delta(\alpha)$) is assumed to be a continuous function with the following properties:

$$(i) 0 \leq \delta(\alpha) \leq 1;\ (ii) \delta(\alpha_0) = 1;\ (iii) \delta'(\alpha) < 0;$$

where α_0 is the primitive technology and $\delta'(.)$ is the first derivative of $\delta(.)$ with respect to α. Therefore, the cost of new technologies is higher for more *capital-intensive* technologies.

2.1 Consumption, savings and bequests

The representative consumer lives two periods and her utility depends on the consumption when young (c), the consumption when old (d) and an intergenerational transfer or bequest (b). We assume zero population growth and a logarithmic utility function, which combines the three arguments (c, d and b). The income of a young consumer is given by the wage w plus the transfer that she gets form her parents. We assume full depreciation, so the return to savings is the interest rate r. The young consumer takes α as given, so the problem for the consumer is the following:

$$\max \left\{ \log(c_t) + \beta \left(\log(d_{t+1}) + \gamma \log(b_{t+1}) \right) \right\} \ s.t.\ w_t + b_t = c_t + \frac{d_{t+1} + b_{t+1}}{r_{t+1}}$$

where β is the discount factor. From this maximization problem we find:

$$c_t = \frac{d_{t+1}}{\beta r_{t+1}} \tag{1}$$

$$b_{t+1} = \gamma d_{t+1} \tag{2}$$

and

$$s_t = (w_t + b_t)\left(\frac{\beta\,(1+\gamma)}{1+\beta\,(1+\gamma)}\right) \tag{3}$$

Since the elasticity of substitution between present and future consumption is equal to one, as is the elasticity of substitution between future consumption and bequest, savings does not depend on the interest rate. This is a standard result in Overlapping Generations Models (see Auerbach and Kotlikoff, 1995). However, two new elements appear. First, α is not a parameter but a variable, and, second, the stock of capital depends not only on savings, but also on the technology. Therefore, factor prices depend not only on factor abundance but also on the technology.

2.2 Technology, factor prices and savings

Agents decide savings when young, so when they are old they take savings as given and maximize utility taking into account that $k_t = \delta(\alpha_t)s_t$.

$$\max_{\alpha_t, l_t}\{(d_t) + \gamma \log b_t\} \;\; s.t. \;\; A\,(\delta(\alpha_t)s_t)^{\alpha_t} - w_t l_t = d_{t+1} + b_{t+1}$$

Therefore, factor prices are determined by savings and technology,

$$w_t = (1-\alpha_t)\,A\,(\delta(\alpha_t)s_t)^{\alpha_t} \quad \text{and} \quad r_t = \alpha_t A\,(\delta(\alpha_t)s_t)^{\alpha_t - 1} \tag{4}$$

The interest rate and the wage are given by the factors' marginal productivity. So α is equal to capital income share and $(1-\alpha)$ is equal to labor income share.

Similarly, the optimal level of α_t is given by the following equality:

$$A(\delta(.)s_t)^{\alpha_t}\left(\ln(\delta(.)s_t) + \delta'(.)\frac{\alpha_t}{\delta(.)}\right) = 0 \tag{5}$$

Note that after paying the cost of technology, the capital labor ratio must be greater than one, that is, $\delta(.)s_t > 1$, since otherwise it is preferable to keep the primitive technology. Rearranging equation 5,

$$\ln s_t = -\delta'(.)\frac{\alpha_t}{\delta(.)} - \ln\delta(.) \tag{6}$$

Deriving equation 6, it is possible to find the relation between the savings, s, and technology, α,

$$\frac{\partial \ln s_t}{\partial \alpha_t} = -\delta'(.) \left(\frac{2}{\delta(.)} \right) + \frac{\alpha_t}{\delta(.)} \left(\left(\delta'(.)\right)^2 \frac{1}{\delta(.)} - \delta''(.) \right) \tag{7}$$

Both the first and the second terms of equation 7 are non-negative, so there exists a positive relation between α_t and s_t.

[Insert figure 1 about here]

From equations 6 and 7, we can define the function $s^{**}(\alpha) = \frac{e}{\delta(.)}^{-\delta'(.)\frac{\alpha_t}{\delta(.)}}$ and plot it in a graph relating s and α (see Figure 1). Depending on the shape of the function $\delta'(.)$ there may exist a minimum level of savings greater than one, such that an increase in α is profitable. Therefore, $s_t > 1$ is a necessary but may not be a sufficient condition for investment in higher qualities of capital to be profitable. A sufficient condition is $s_t > s^{**}(\alpha_0)$.

We can summarize the previous results in Proposition 1:

Proposition 1 *(i) $s_t > 1$ is a necessary condition for an increase in capital intensity (and capital income share) to be profitable. (ii) $s_t > s^{**}(\alpha_{t-1})$ is a sufficient condition for an increase in capital intensity (and capital income share) to be profitable (iii). In capital abundant economies ($s_t > s^{**}(\alpha_0)$), there exists a positive relation between α_t and s_t.*

On the one hand, given the technology, the production function is concave and satisfies the Inada conditions, in particular $\lim_{s \to 0} \frac{\partial y}{\partial s} = \infty$. On the other hand, the marginal productivity of a unit of savings invested in new technologies positively depends on the amount of savings. Therefore, for low savings levels it is better not to invest in capital intensive technologies. Now, if the savings level is high enough to make technological changes profitable, then there exists a positive relation between savings and technologies because the gains derived from adopting capital intensive technologies positively depend on the level of savings.

2.2.1 Savings

For simplicity, we assume that the population is equal to 1 for each generation, so total output is Ak_t^{α}. The output must be divided between young people (w) and old people (rk). Old people consume and leave a bequest ($rk = b + d$). Therefore,

$$A(\delta_t(\alpha_t)\, s_t)^{\alpha_t} = w_t + b_t + d_t \tag{8}$$

Combining with equations 2 and 4, and rearranging, yields

$$b_t = \left(\frac{\gamma}{1+\gamma}\right) \alpha_t A \delta_t(\alpha_t)\, s_t{}^{\alpha_t}$$

so equation 3 can be written as

$$s_{t+1} = A\left(\delta_t(\alpha_t)\, s_t\right)^{\alpha_t}(1+\gamma-\alpha_t)\left(\frac{\beta}{1+\beta(1+\gamma)}\right) \tag{9}$$

Therefore, the savings level at time $t+1$ is fully determined by the state variables of time t. This result, which depends on the assumed utility function, greatly simplifies the analysis. Indeed, since in the optimum technology is a function of savings, technology at time $t+1$ is also fully determined by the state variables at time t.

Now, using equation 9 we derive the growth rate of savings,

$$\frac{s_{t+1}}{s_t} = (1+\gamma-\alpha_t))\, A\left(\delta(\alpha_t)\right)^{\alpha_t} s_t{}^{\alpha_t-1}\left(\frac{\beta}{1+\beta(1+\gamma)}\right)$$

Therefore, we can find a steady state level of savings given the technology:

$$s_{ss}(\alpha) = \left(\frac{(1+\gamma-\alpha)\,\beta A\delta(.)^{\alpha}}{1+\beta(1+\gamma)}\right)^{\frac{1}{1-\alpha}} \tag{10}$$

Now, to have an idea of the shape of $s_{ss}(\alpha)$ we take logs and derivatives,

$$\frac{\partial \ln s_{ss}}{\partial \alpha} = \frac{1}{1-\alpha}\left(\frac{1}{1-\alpha}\ln\frac{(1+\gamma-\alpha)\,\beta A\delta(.)^{\alpha}}{1+\beta(1+\gamma)} + \left(\alpha\frac{\delta'(.)}{\delta(.)} - \frac{1}{1+\gamma-\alpha}\right)\right)$$

$$\frac{\partial \ln s_{ss}}{\partial \alpha} = \frac{1}{1-\alpha}\left(\ln s_{ss} + \left(\alpha\frac{\delta'(.)}{\delta(.)} - \frac{1}{1+\gamma-\alpha}\right)\right)$$

Note that the second term in parenthesis is always negative. So for low levels of s_{ss} the slope is negative, but for high levels of s_{ss} the slope of $s_{ss}(\alpha)$ is positive. Therefore, both the level of s_{ss} for a given α and the slope of $s_{ss}(\alpha)$ depend of TFP, A, preference for bequest, γ, the discount factor, β, and the cost of technologies, $\delta(.)$.

Finally, from equation 10 it follows that (i) if $\frac{\gamma\beta A\delta(.)^{\alpha}}{1+\beta(1+\gamma)} < 1$, then the function $s_{ss}(\alpha)$ is bounded from above and (ii) $\frac{\gamma\beta A\delta(.)}{1+\beta(1+\gamma)} > 1$ for any α is a necessary condition for the economy to have long-run growth. We present this result in Proposition 2.

Proposition 2 $\lim_{\alpha\to 1}\delta(\alpha) > \frac{1}{A}\frac{1+\beta(1+\gamma)}{\gamma\beta}$ *is a necessary condition for the economy to present long-run growth.*

Therefore, bequests must be positive ($\gamma > 0$) and TFP high, otherwise long-run growth is not possible. If there are no bequests ($\gamma = 0$), Proposition 2 indicates that long-run growth is not possible[1]. This result was previously obtained by Boldrin (1992) and Jones and Manuelli (1992) in different set-ups. Additionally, Proposition 2 tell us that for the economy to reach long-run growth, the cost of capital goods must be bounded ($\lim_{\alpha\to 1}\delta(\alpha) > 0$).

We explain the intuition behind Proposition 2 in the following lines:

First, savings at time $t+1$ are the product of the savings rate and the output at time t. So given the savings rate, the higher the TFP, the higher the savings level. For any initial conditions, if TFP is low, savings are low, and long-run growth is not possible.

Second, since output is divided between workers and capital owners and the latter do not save, the savings rate of the economy depends on the labor income share. Thus, if the technology continuously grows more capital intensive, labor income share decreases period by period. If bequests are zero, then the savings rate decreases as the technology becomes more capital intensive and converges to zero as α goes to one. Therefore, if there are no bequests long-run growth is not possible.

[1]This result depends on the assumption of Cobb-Douglas production function and on the OLG setting.

Third, if $\lim_{\alpha\to 1}\delta(\alpha)=0$ the cost of capital goods goes to infinity as α goes to one, so the incentives to adopt more capital intensive technology disappear.

2.3 Steady state and long-run growth

Until now we have described the behavior of capital abundant economies, where agents have incentives for technical changes and the behavior of low income economies, where there are no incentives for technical changes. In this section we identify the conditions under which an economy can achieve long-run growth and characterize the steady state, whenever it exists.

To characterize the steady state we can use the functions $s^{**}(\alpha)$, which shows the relation between technology and savings that old agents choose, and $s_{ss}(\alpha)$, which shows the steady sate level of savings given the technology. Using these functions it is possible to find a steady state level of technology, α_{ss}, defined by $s_{ss}(\alpha_{ss})=s^{**}(\alpha_{ss})$. Note that, if $s=s^{**}(\alpha_{ss})$, then the chosen technology is α_{ss} and if the technology is α_{ss} then the steady state level of savings is $s_{ss}(\alpha_{ss})$. Therefore, α_{ss} is a steady state technology.

In figures 2 and 3 we plot the functions $s^{**}(\alpha)$ and $s_{ss}(\alpha)$. In figure 2 we assume that the slope of $s_{ss}(\alpha)$ is negative (low levels of A, γ and β). In this case the two functions cross only once, so there is one steady state technology (α_{ss}) and one steady state savings level ($s_{ss}(\alpha_{ss})$). Note also that the steady state is stable, that is, if $s>s_{ss}(\alpha_{ss})$, then the growth rate of savings is negative and if $s<s_{ss}(\alpha_{ss})$, then the growth rate of savings is positive.

In figure 3 we assume that the slope of $s_{ss}(\alpha)$ is positive and converges to infinity when α goes to one (high levels of A, γ and β and $\lim_{\alpha\to 1}\delta(\alpha)>\frac{1}{A}\frac{1+\beta(1+\gamma)}{\gamma\beta}$). In this case we may have two steady states, one stable, α_{ss}, and one unstable, α^*. However, we may also have no steady state. Indeed, any increase in TFP moves up the function $s_{ss}(\alpha)$, augmenting α_{ss} and reducing α^*. Therefore, there exists a level of TFP, A^{**}, such that $\alpha_{ss}=\alpha^*$. From the definition of A^{**} it follows that if $A>A^{**}$, there is no steady state.

[Insert figures 2 and 3 about here]

Proposition 3 summarizes the previous results.

Proposition 3 *If* $\lim_{\alpha \to 1} \delta(\alpha) > \frac{1}{A} \frac{1+\beta(1+\gamma)}{\gamma\beta}$ *and* $A < A^{**}$, *then there exists a level of technology,* α^{*}, *such that: (i)* $s_{ss}(\alpha^{*}) = s^{**}(\alpha^{*})$, *(ii) if under the initial conditions the agents of the economy choose a technology* α *such that* $\alpha < \alpha^{*}$, *the economy converges to a steady state and (iii) if under the initial conditions the agents of the economy choose a technology* α *such that* $\alpha > \alpha^{*}$, *the economy has long-run growth.*

Thus, minimum level of savings, $s^{**}(\alpha^{*})$ is needed to achieve long-run growth, so there may exist a poverty trap.

The productivity of new technologies depends on the savings level and the productivity of savings depends on the capital intensity of the technology. Therefore, if savings are small it is not worth to invest in capital intensive technologies and if the technology is labor intensive the effect of savings on output is small. Thus, if the initial conditions are such that the technology is labor intensive and the savings level is small, then the economy can be in a poverty trap. However, if the initial conditions are such that the savings level is high, then it is optimal to adopt capital intensive technologies and the effect of savings on output is big. Thus, if the initial conditions are such that the technology is capital intensive and the savings level is high, the economy presents long-run growth.

Propositions 1, 2 and 3 tell us that capital abundant economies have incentives to increase α and for these economies any increase in α generates increases in output and savings, yielding a virtuous cycle of technological change and economic growth. This process of economic growth may last forever if TFP and bequests are high and if the cost of capital goods is bounded. The other side of the story is that when TFP and bequests are not high enough, poor economies are trapped in a steady state.

In summary, we have presented an OLG model, where any technology can be adopted paying a cost. Markets are competitive and marginal productivity of factors determines factor prices. Therefore, innovations affect factors income share. As a result, the elasticity of output with respect to reproducible factors and the income share of such factors (physical and human capital) depend on the factor abundance of the economies. Under this framework, depending on the initial conditions, in some economies the production function converges to AK, while in other economies, long-run growth is zero.

2.4 Dynamics of the model

So far we have presented the model, characterized the steady state and described the possible equilibrium paths including long-run growth and a poverty trap. In this section we combine the production function, the savings rate and the relation between technology and savings to illustrate the dynamics of the model in a graphical way.

We use the inverse of the function $s^{**}(\alpha)$ to express technology, α, as a function of savings, $\alpha(s)$.

The production function is given by $Y = AK^{\alpha}L^{1-\alpha}$. Therefore, if $s < s^{**}(\alpha_0)$, then $\frac{\partial Y}{\partial s} = \alpha_0 A\,(s_t)^{\alpha_0 - 1}$ and $\frac{\partial^2 Y}{\partial s^2} = (\alpha_0 - 1)A\,(s_t)^{\alpha_0 - 1} < 0$, namely the function is concave in savings. However, if $s > s^{**}(\alpha_0)$, then

$$\frac{\partial Y}{\partial s} = \alpha(.)A\,(\delta(.)s_t)^{\alpha(.)-1}\left(\delta(.) + \delta'(.)\alpha'(.)s_t\right) + A\,(\delta(.)s_t)^{\alpha(.)}\ln(\delta(.)s_t)\alpha'(.)$$

Rearranging,

$$\frac{\partial Y}{\partial s} = A\,(\delta(.)s_t)^{\alpha(.)}\left(\frac{\alpha(.)}{s_t} + \alpha'(.)\left(\alpha(.)\frac{\delta'(.)}{\delta(.)} + \ln(\delta(.)s_t)\right)\right)$$

From equation 6 it follows that $\alpha(.)\frac{\delta'(.)}{\delta(.)} + \ln(\delta(.)s_t)$, so

$$\begin{aligned} \frac{\partial Y}{\partial s} &= \alpha(.)A\,(\delta(.)s_t)^{\alpha(.)-1}\,\delta(.) \\ \text{and } \lim_{\alpha\to 1}\frac{\partial Y}{\partial s} &= A\lim_{\alpha\to 1}\delta(.) \end{aligned}$$

Thus, if $\lim_{\alpha\to 1}\delta(.) > 0$, then in the long-run, the production function converges to a linear function of type AK. Therefore, we can plot production and savings in a graph (see figure 4). The bold line is the production function. It is concave for low levels of savings and as the savings grow, the slope of the function gets closer to a constant $(A\lim_{\alpha\to 1}\delta(.))$.

[Insert figure 4 about here]

The savings rate is given by $(1+\gamma-\alpha_t)\left(\frac{\beta}{1+\beta(1+\gamma)}\right)$, so if $s < s^{**}(\alpha_0)$, it is constant and if $s > s^{**}(\alpha_0)$ it decreases as α grows (see figure 5) and converges to $\left(\frac{\gamma\beta}{1+\beta(1+\gamma)}\right)$ as α goes to one.

[Insert figure 5 about here]

Finally, given the behavior of the savings rate, we can plot the level of savings at time $t+1$ as a function of the level of savings at time t (see figure 6). When savings at time t are smaller than $s^{**}(\alpha_0)$, savings are a constant proportion of output (concave). When $s_t > s^{**}(\alpha_0)$, the savings rate decreases as savings grow, so the growth of savings is slower. However, the savings rate is bounded from below and in the limit it is equal to $\frac{\gamma\beta}{1+\beta(1+\gamma)}$, so as the savings at time t converge to infinity, the savings rate converge to a constant and the growth rate of savings becomes equal to the growth rate of output. That is, the slope of the savings function becomes constant in the long-run. The dashed line in figure 6 is the 45° line, so when the savings function is above the dashed line, the growth rate of savings is positive and when it is below the dashed line, the growth rate of savings is negative.

[Insert figure 6 about here]

In figure 6 we observe that depending on the initial level of savings, in some economies the production function converges to AK, while in other economies, long-run growth is zero.

3 Discussion

3.1 The relative price of capital goods

In the previous section we assumed that the representative agent uses her savings to build a capital good and chooses the quality of the capital good maximizing her utility. We can also assume that agents give their savings to a capital good producer and receive a capital good of the desired quality. According to our model, the price of a capital good which has embodied technology α would be given by,

$$p = \frac{1}{\delta(\alpha)}$$

So, the price of a capital good depends positively on the capital intensity of the technology. Therefore, as an economy grows and uses more capital intensive technologies, the relative price of capital good augments. This result contrasts with the empirical evidence. Indeed, the relative price of capital goods has not an increasing trend but very much the opposite, it has decreased in the last few decades. One possible explanation for this contradiction is that the productivity in the production of capital goods has increased, moving up the function $\delta(\alpha)$ and reducing the relative price of capital for every quality. If this were the case, the price of capital goods of higher qualities would be higher, but the relative price of the capital goods with respect to the consumption good would not grow with the stage of development.

Modelling changes in productivity in the production of capital goods goes beyond the aim of this paper. However, these changes have important implication for the model we are considering. In particular, if the production of capital goods experiences a positive shock in productivity, then the price of capital goods decreases. Therefore, a higher α can be obtained without paying additional costs.

In the next subsection we consider the possibility of positive cost-less shocks in technology. This would be the case of a developing economy, where agents take the price of capital goods as given and experience an exogenous decrease in such prices. We model this shock assuming that a more capital intensive technology is available but $\delta(.)$ remains constant. For simplicity we take $\delta(.) = 1$.

3.2 Exogenous shocks, factor prices and savings

3.2.1 Factor Prices

The choice of capital intensive technologies is made by the elderly. If there is no coordination among the elderly, the firms take the wage as given and decide on the technology. In this case they increase capital intensity only if the capital labor ratio is higher than one ($s > 1$). If it is lower than one, an increase in α reduces the output and, given the wage, reduces also the capital income. However, even if the capital labor ratio is higher than one, the effect of new technologies may reduce workers income.

Taking logs and derivatives in equation 4 we can find the effect that a change in α has on the wage and on the interest rate,

$$\frac{\partial \ln w_t}{\partial \alpha_t} = -\frac{1}{1-\alpha_t} + \ln(s_t) \tag{11}$$

$$\frac{\partial \ln r_t}{\partial \alpha_t} = \frac{1}{\alpha_t} + \ln(s_t) \tag{12}$$

Therefore, if $s_t > e^{-\frac{1}{1-\alpha_t}}$, an exogenous increase in the capital intensity of the technology generates an augment in wages and if $s_t > e^{-\frac{1}{\alpha_t}}$, an exogenous increase in the capital intensity of the technology generates an augment in interest rates.

The intuition behind the previous results can be better understood considering the effects of an increase in α one by one: (i) an increase in α positively (negatively) affects output, whenever the capital labor ratio is greater (smaller) than one ($s_t > 1$); (ii) an increase in α augments the capital income share; (iii) an increase in α reduces the labor income share; and (iv) the higher the capital labor ratio, the higher the effect that an increase in α has on output.

The wage is the product of labor income share and output. Therefore, from (i) and (ii), if the capital labor ratio is lower than one, any increase in α reduces wages. If the capital labor ratio is higher than one, the effect that an increase in α has on the wages can be positive (negative) if the increase in output is bigger (lower) than the decrease in the labor income share. From (iv) it follows that the effect that an increase in α has on the wages can only be positive when the savings level is very high.

The interest rate is the product of capital income share and output. Therefore, if the capital labor ratio is higher than one, any increase in α increases interest rates. If the capital labor ratio is lower than one, the effect that an increase in α has on the interest rate can be positive (negative) if the decrease in output is smaller (bigger) than the increase in the capital income share. From (iv), it follows that an increase in α can only reduce the interest rate when the savings level is very low.

In summary, it is clear that technology affects output and income distribution. Since the choice of technologies is made by the capital owners, the chosen technology is not likely to be the one that maximizes output or economic growth. This fact may have interesting implications for economic policy. However, a more complex setting with more dimensions of heterogeneity would be needed to propose serious policy recommendations.

3.2.2 Savings

Young people's income has two components, wages and bequests. On the one hand, any change in technology that increases capital income also increases the bequest. On the other hand, we know that an increase in α may increase or decrease wages depending on the level of savings per worker. Thus, the net effect of a change in technology on capital accumulation can be positive or negative. To find the effect that an exogenous shock in the capital intensity has on future savings, we take logs and derivatives in equation 9 (assuming $\delta(.) = 1$),

$$\frac{\partial \ln(s_{t+1})}{\partial \alpha_t} = -\frac{1}{1+\gamma-\alpha_t} + \ln(s_t) \tag{13}$$

and define the function $s^*(\alpha)$ by the equality $s^*(\alpha) = e^{-\frac{1}{1+\gamma-\alpha_t}}$. Therefore, the result in equation 13 can be summarized in proposition 4.

Proposition 4 *For any α there exists a minimum amount of savings $s^*(\alpha)$, such that for any $s_t > s^*(\alpha)$ an exogenous increase in current α generates an increase in future savings and any $s_t < s^*(\alpha)$ an exogenous increase in current α generates a decrease in future savings*

To have a better intuition of the previous results, consider the effect that an increase in α has on wages and bequests. The bequest is the product of capital income share, output and the parameter γ. Therefore, if the capital labor ratio is greater than one, any increase in α increases the bequest. Now, recall that the effect of a technological change on the wage can be positive or negative depending on the capital labor ratio. Since the components of the income of a young person are the wage and the bequest, the net result of an increase in α depends on the capital labor ratio (s); if it is very high, both the wage and the bequest increase when a more capital intensive technology is adopted and, by the same token, the income of young people as well as their savings increase. If the capital labor ratio is not high enough, wages decrease and the income of young people can increase or decrease depending on the behavior of the bequests.

Finally, using proposition 4 and the fact that whenever the capital labor ratio is higher than one, the elderly have incentives to adopt capital intensive technologies, Proposition 5 follows directly:

Proposition 5 *Given α, if $s^*(\alpha_{t-1}) > s_t > 1$ then: (i) Capital owners have incentives to invest in new technologies, that is, increase α and (ii) any increase in α reduces future savings.*

Therefore, for some economies, agents choose to adopt more capital intensive technologies and, as a result, savings decrease as well as the incentives to further technological change. In these economies, the technological change leads to a reduction in future per capita income.

4 A numerical example

In this section we consider an explicit functional form for the cost of technologies and run some simulations for different parameter values in order to illustrate the dynamics of the model in a comprehensive way. We choose the function $\delta(\alpha) = \min[1, \phi(\chi - \alpha)]$ so, by equation 6 the relation between savings and technology is given by $s^{**}(\alpha) = \frac{e^{-\frac{\alpha}{\phi(\chi-\alpha)}}}{\phi(\chi-\alpha)}$. Therefore, given the initial level of savings, we get the technology, and consequently the output, wages, bequests and savings for the next period. In other words, with the parameters of the model and the initial level of savings we can describe the dynamics of the system.

Table 1 presents eight scenarios with different parameter values (in all of them we assume $\beta = 1$). In the first four scenarios we take the same values for all parameters but for TFP ($\alpha_0 = 0.2$, $\gamma = 0.8$, $\chi = 2.4$, $\phi = 0.5$). In scenarios 5 to 8, we assume different parameters for the function $\delta(\alpha)$ and the given technology ($\alpha_0 = 0.1$, $\chi = 5.33$, $\phi = 1.5$) and allow for changes in both TFP (A) and the preference for bequests (γ). In scenarios 5 and 6 we keep TFP constant ($A = 3.15$) but change the preference for bequests (γ). In scenarios 7 and 8 we change both A and γ.

[Insert Table 1 about here]

Table 2 presents the main results of the simulations. Column 1 contains the minimum level of savings, such that the agents have incentives to invest in technologies better than α_0. Columns 2 and 3 show the steady state levels of savings and technology and columns 4 and 5 show the minimum levels of savings and technology needed to achieve long-run growth.

In scenarios 1, 2 and 5 there exists a steady state but also the possibility for long-run growth if the initial level of savings is high enough ($s > s_{ss}(\alpha^*)$), namely, there is a poverty trap (see Proposition 3). In scenarios 3, 6 and 7 there exists a unique steady state and there is no possibility for long-run growth ($\lim_{\alpha \to 1} \delta(\alpha) < \frac{1}{A} \frac{1+\beta(1+\gamma)}{\gamma\beta}$). Finally, in scenarios 3 and 7 there is no steady state and every economy presents long-run growth independently of the initial conditions ($A > A^{**}$).

The main conclusions of these simulations confirm what we found in the previous section: (i) the possibility of long-run growth depends on the levels of TFP and bequests, (ii) increasing TFP reduces the maximum amount of savings leading to a poverty trap and eventually may eliminate the trap, and (iii) increasing the bequest reduces the maximum amount of savings leading to a poverty trap and eventually may eliminate the trap.

[Insert Table 2 about here]

To see how the initial level of savings can affect the path of the economy, we take scenario 5 and simulate the dynamics using different initial levels of savings.

[Insert Figures 7 to 10 about here]

Figure 7 relates the technology α_t with the change in savings, $s_{t+1} - s_t$. Since there exists a unique relation between α_t and s_t, this plot can be done with s instead of α. In any case, from the figure it follows that:

(i) In economies where the initial conditions are such that $\alpha_0 = 0.576$ ($s_0 = 2.5$), the growth of savings is zero, so these economies are in a steady state.

(ii) In economies where the initial conditions are such that $\alpha_0 < 0.576$ ($s_0 < 2.5$), the growth of savings is positive but decreasing, so these economies converge to a steady state.

(iii) In economies where the initial conditions are such that $0.576 < \alpha_0 < 0.815$ ($2.5 < s_0 < 3.12$), the growth of savings is negative, so these economies converge to a steady state.

(iv) In economies where the initial conditions are such that $0.815 < \alpha_0$ ($3.12 < s_0$), the growth of savings is positive and increasing, so these economies present long-run growth.

Figures 8 to 9 show simulations of the dynamics of output, wages, bequests and savings, starting from the conditions described in (ii), (iii) and (iv) -time in the horizontal ax-. Figure 8 shows the behavior of output, wages, bequests and savings when $s_0 = 1.6$. In this case, output and bequests grow while wages decrease. The former effect is bigger than the latter, so savings grow and technology becomes more capital intensive. However, the rate of change of the different variables is reduced period by period, until the point where the economy achieves a steady state.

Figure 9 shows the behavior of output, wages, bequests and savings when $s_0 = 2.6$. In this case, output and bequests decrease while wages grow. The former effect is bigger than the latter, so savings decrease and technology becomes more labor intensive. Again, the rate of change of the different variables is reduced period by period until the point where the economy achieves a steady state.

Figure 10 shows the behavior of output, wages, bequests and savings when $s_0 = 3.2$. In this case output and bequests grow while wages decrease. The former effect is bigger than the latter, so savings grow and technology becomes more capital intensive. Finally, the change of the different variables is increasing period by period, so the economy presents long-run growth.

5 Conclusions

Traditionally, growth theory considers innovations as changes in TFP. Here we consider the possibility of changing the factor intensity of the technology. Using this framework, we find that capital abundant countries have incentives to make *labor-saving* innovations, while labor abundant economies do not have incentives to innovate.

If TFP and bequests are high enough, increases in capital share raise the future capital labor ratio. As the capital labor ratio increases, α grows too. Thus, for initially capital abundant economies, both the capital labor ratio and the capital share grow. Depending on the cost of new technologies the production function of these economies may converge to AK, so they can have permanent growth even without changes in TFP. If bequests and TFP are small, there is no possibility of long-run growth because technological change reduces wages and capital accumulation.

For economies where the capital labor ratio is low, there are no incentives to innovate. Nevertheless, if their steady sate capital is high enough, at some moment in the future the incentives to change technology appear. Depending on the TFP and bequests, these economies can be trapped in a steady state or can reach a capital labor ratio high enough to start making innovations.

Labor-saving innovations affect output and income distribution. Since the choice of technologies is made by the capital owners, the chosen technology is not likely to be the one that maximizes output or economic growth. Indeed, some innovations reduce labor income as well as savings and, as a consequence, economic growth decreases. This fact may have implications for economic policy. However, a more complex model, with more dimensions of heterogeneity, would be needed to propose serious policy recommendations.

The model we have presented here is a simplification that helps to identify some of the implication of *capital-using* and *labor-saving* of innovations. For future research it can be interesting to analyze the effects of these innovations in a setting where assets are randomly distributed among agents and workers can be capital owners. Other extension can be related to the fiscal policy implications of this type of technological change.

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Tables and Figures

Table 1: Parameters

	α_0	γ	A	χ	ϕ
Scenario 1	0.2	0.8	2.8	2.4	0.5
Scenario 2	0.2	0.8	3.2	2.4	0.5
Scenario 3	0.2	0.8	2.0	2.4	0.5
Scenario 4	0.2	0.8	5.0	2.4	0.5
Scenario 5	0.1	0.8	3.15	5.33	0.15
Scenario 6	0.1	0.5	3.15	5.33	0.15
Scenario 7	0.1	0.5	3.5	5.33	0.15
Scenario 8	0.1	0.65	4.0	5.33	0.15

Table 2: Results

	$s^{**}(\alpha_0)$	$s_{ss}(\alpha_{ss})$	α_{ss}	$s_{ss}(\alpha^*)$	α^*
Scenario 1	1.11	2.6	0.56	199	0.998
Scenario 2	1.11	4.5	0.69	62	0.951
Scenario 3	1.11	1.3	0.30	-	-
Scenario 4	1.1	-	-	-	-
Scenario 5	1.69	2.50	0.576	3.12	0.815
Scenario 6	1.69	1.821	0.199	-	-
Scenario 7	1.69	2.013	0.323	-	-
Scenario 8	1.69	-	-	-	-

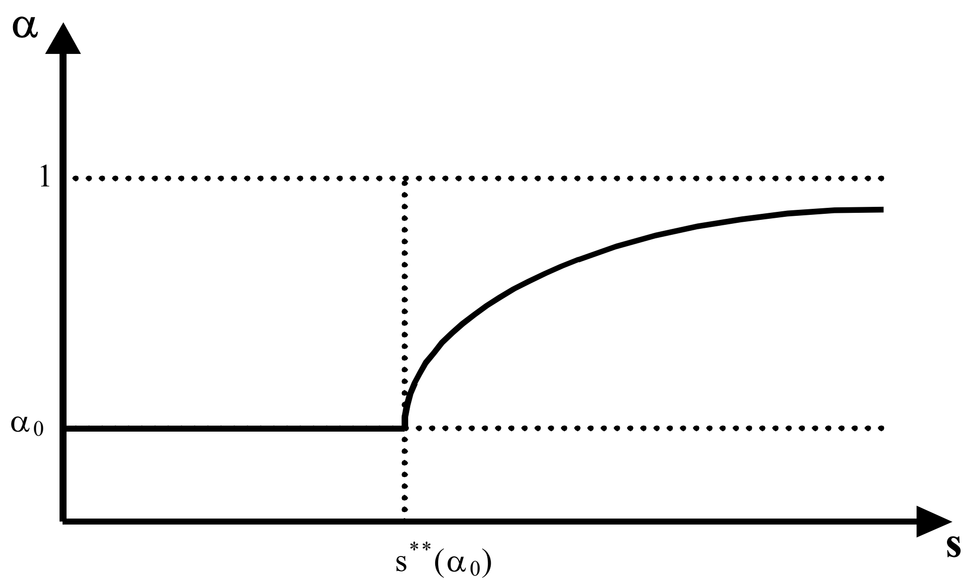

Figure 1: Technology and Savings

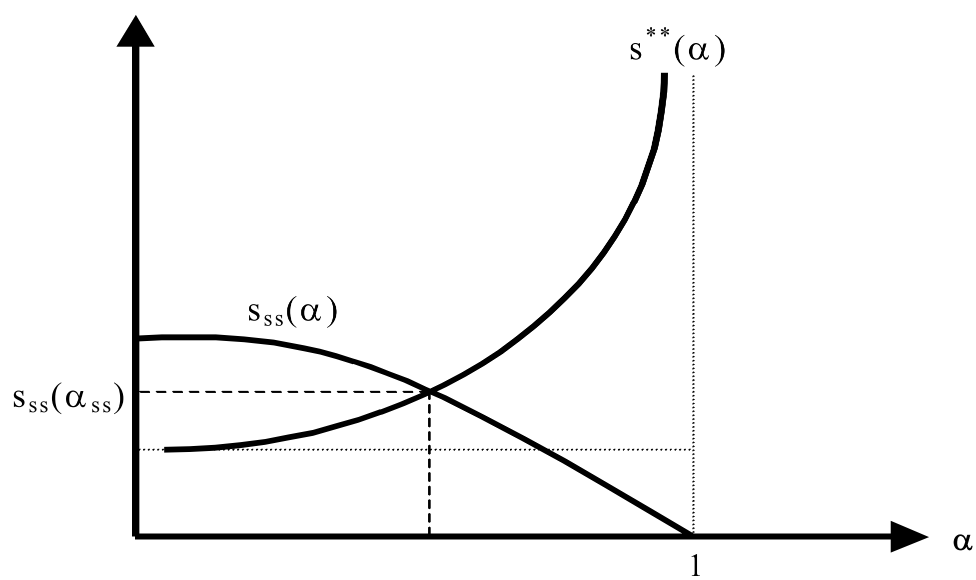

Figure 2: Steady State

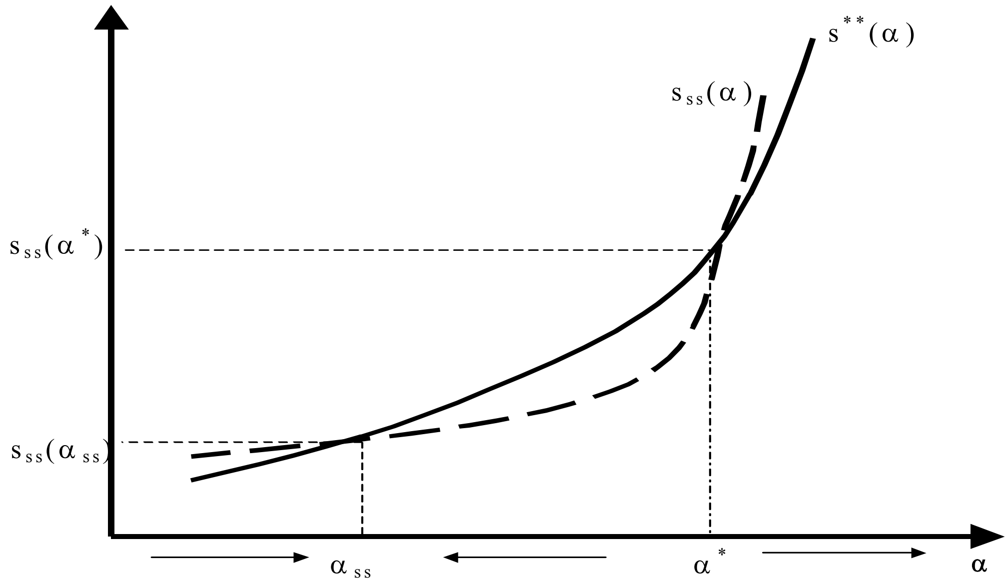

Figure 3: Steady State and Long-run Growth

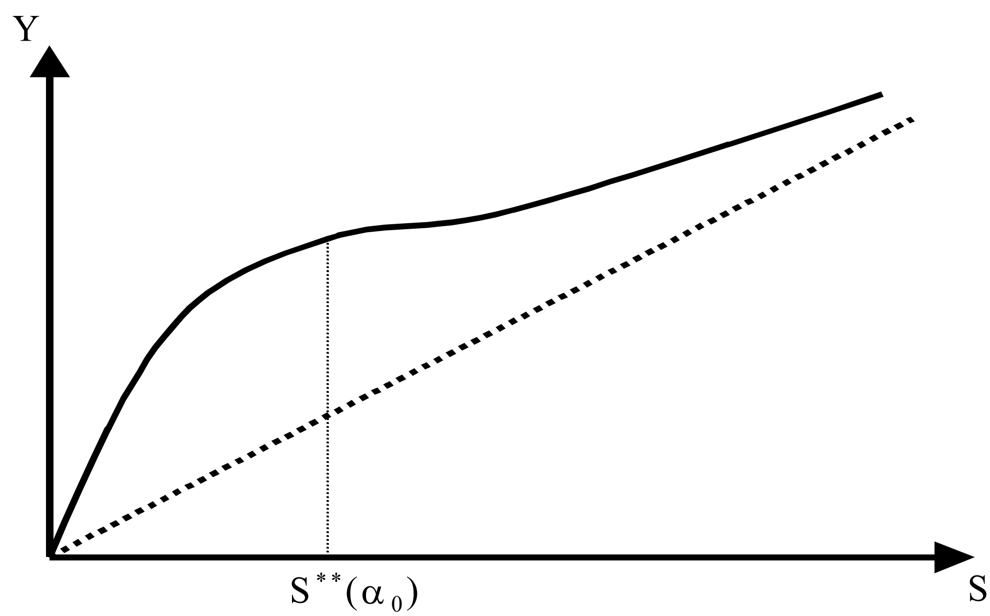

Figure 4: Production Function

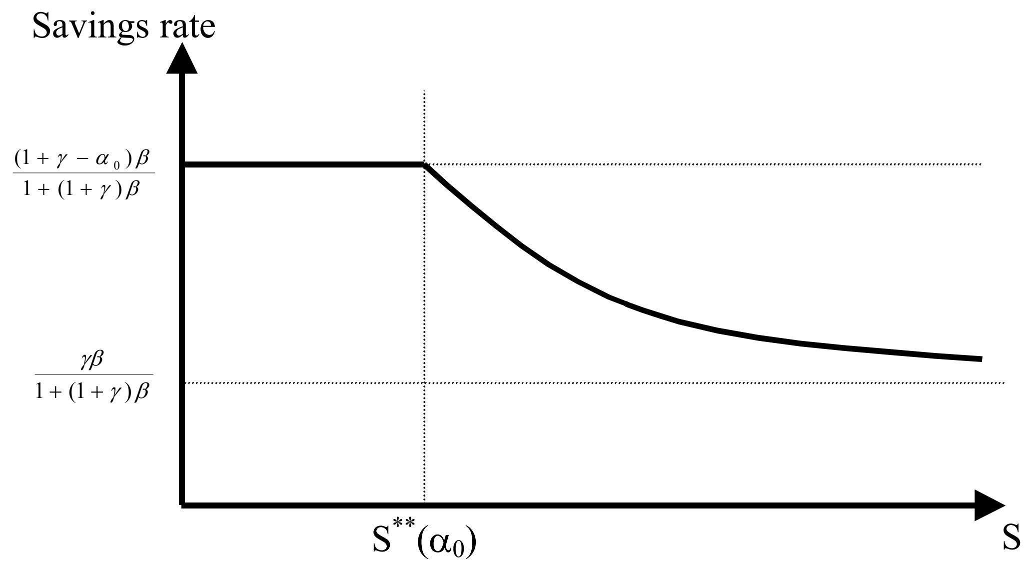

Figure 5: Savings and Savings Rate

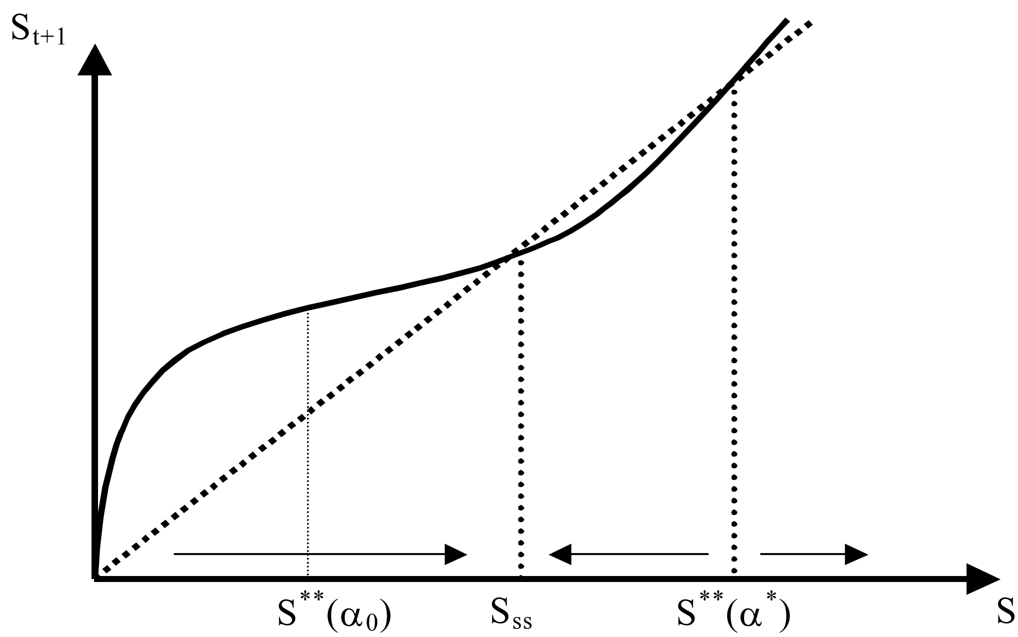

Figure 6: Dynamics of Savings

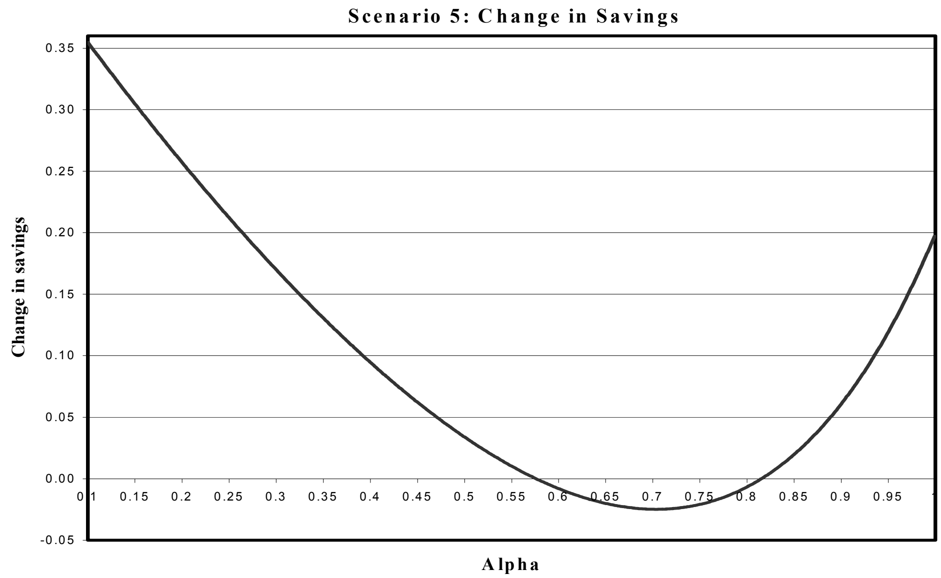

Figure 7: Technology and Savings Growth

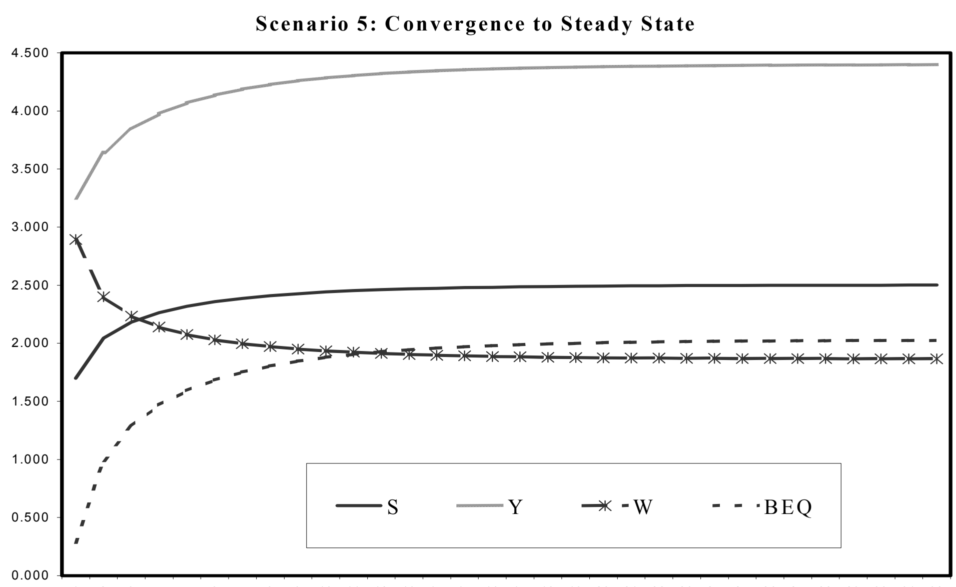

Figure 8: Income, Wages, Bequests and Savings.

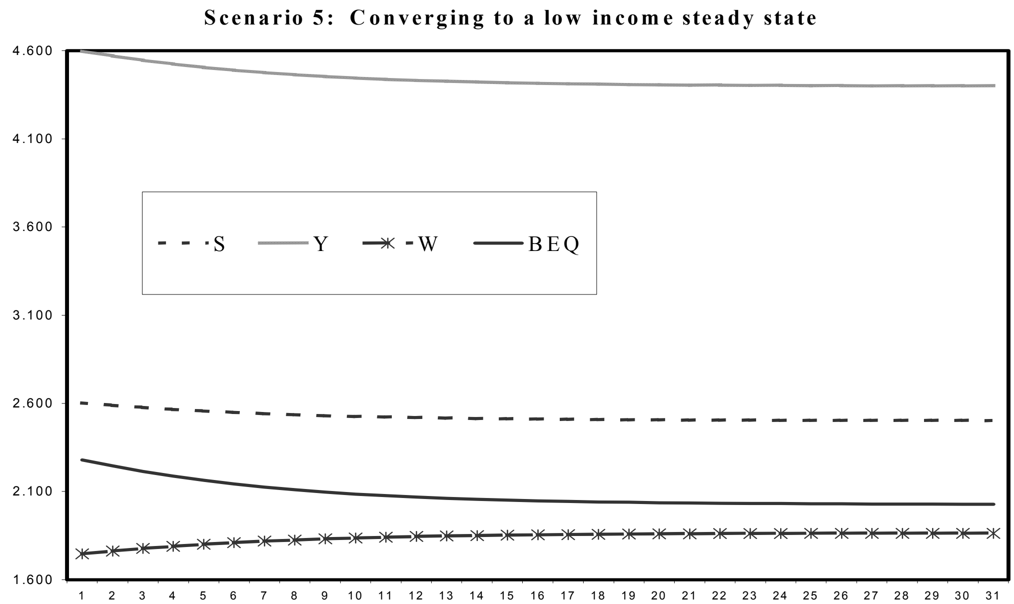

Figure 9: Income, Wages, Bequests and Savings

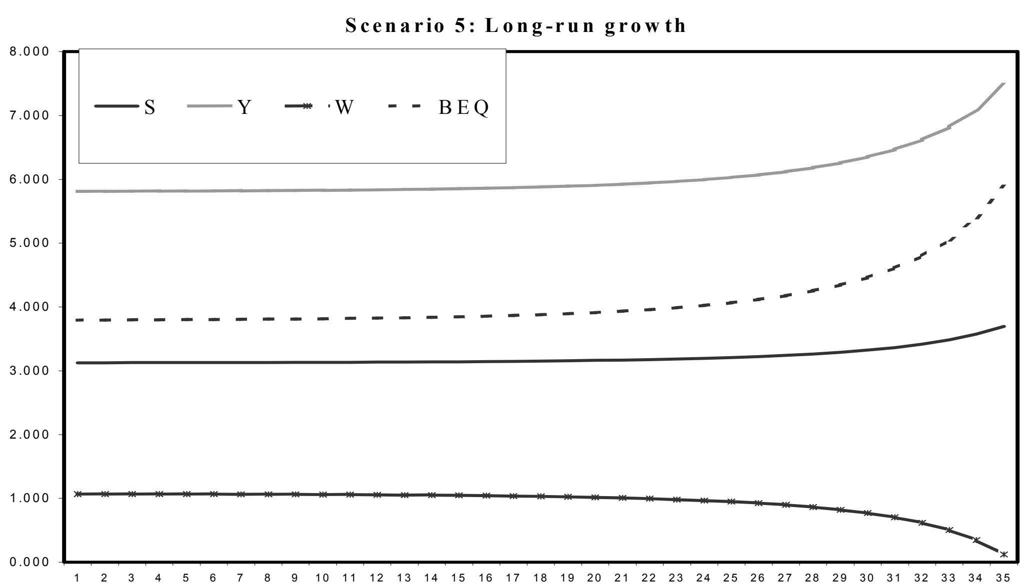

Figure 10: Income, Wages, Bequests and Savings