Welfare Gains from Withdrawing Consumption Risk: Measuring the Benefits Distribution from the Public Health Insurance Expansion in Mexico

Jorge O. Moreno*
ITAM - School of Business Administration

February 8, 2011

Abstract

In April 2003 the Mexican Congress approved a large reform on the provision and financing of public health services. The target population of this policy are those households that were not covered by previous health insurance welfare and which, according to the Ministry of Health, in 2002 accounts for almost 55 million households.

This paper present a proposal for measuring the welfare gains from the expansion in public health insurance policy. For this purpose, I estimate the distribution of gains from reducing the risk in the net consumption the households face after the policy is implemented. In this case, a change in the relative risk of the households due to health insurance coverage is similar to a subisity in the relative price of the risk faced by the households otherwise.

Using preliminary results based on the Mexican National Household Income Expenditure Survey (ENIGH) for 2004, I present evidence that suggest large differences in the relative consumption risk between insured and uninsured households within each decile, and across deciles; particularly the data shows that household lead by women at the bottom of the income distribution are relatively more vulnerable to out of pocket expenditure shocks, and present a relative higher mean and variance when measured as percentage of their income.

Finally I perform a calibration exercise to test the implications of the model for the expected gains under different mean-variance specifications for a typical low income household. For a household with risk aversion of 1 and facing a health expenditure shock with mean-variance of 8-25 dollars per quarter, the insurance policy would imply a consumption gain of 2.5 percent and the willigness to pay would be 3.5 dollars per quarter.

This proposal complements other approaches such as natural experiments or dynamic general equilibrium and provides new insights toward a more complete evaluation of the welfare implications of this public policy.

*ITAM - School of Business Administration, Rio Hondo No. 1, Col. Progreso Tizapan, Mexico D.F. 01080, Mexico. Email: jorge.moreno@itam.mx.

1 Introduction

> *The welfare case for insurance of all sorts is overwhelming. It follows that the government should undertake insurance in those cases where this market, for whatever reason, has failed to emerge. Kenneth Arrow (1963)*

According to a recent study carried out by the Mexican Ministry of Health, in 2002 almost 54.7 percent of the Mexican households were not covered by any health insurance service. On the other hand, from their formal insurance-covered counterpart, 42.6 percent were insured through the national social security system (Instituto Mexicano del Seguro Social, IMSS[1]), 0.5 per cent used private insurance companies, and only 1.5 per cent were insured both by private and IMSS.

When a catastrophic health shock occurs, lack of health insurance protection translates into direct out of pocket expenditures in doctor visits, medicines, and hospital services. Given that a large percentage of the people vulnerable to health income shocks are concentrated in the lower income deciles, starting January 2004 the Mexican Government implemented a large welfare program named "Seguro Popular." This welfare program is a public health insurance (PHI from now) with two main objectives: 1) improve the access to basic health services and financial protection against *out of pocket* expenditure in health services for the poorest population; and 2) change the efficiency and equity in the public financing (subsidies) and the public provision of health services. The goal of PHI is to reach the Mexican population not covered by the conventional and formal social security network by year 2010, and the target population include households lead by: i) self-employed; ii) unemployed; iii) workers of the informal sector; or iv) are outside the (formal) labor market.

PHI is a welfare program mostly financed by both the States and Federal governments and provides full coverage on a basic health services plan. However some small burden of this insurance will be faced by the household by a familiar annual fee and the participation decision. The annual fee payment would be calculated according to the household level of income and earnings and this service will be available only to those families not covered by any kind of insurance. PHI guarantees access to 154 health care interventions, such as preventive and curative health care services with their respective prescriptions; this service covers more than 90 percent of the causes of outpatient medical services of the public medical institutions. Moreover, PHI also provides protection against catastrophic expenditures faced by insured families due to other expensive health diseases.

[1]IMSS is actually financed by the workers in the formal sector, their employers, and the Mexican Government. Formal workers are the only with access to IMSS interventions at reduced, almost zero, costs. Those people not covered by IMSS requires to pay for any intervention, and the prices they face are closer to the competitive ones offered in other private institutions.

Figure 1: Mexico's Seguro Popular Fee Structure by Income Decile */

Decile	Fee US$2004
I	0
II	0
III	57
IV	111
V	165
VI	225
VII	289
VIII	448
IX	596
X	903

*/ MX$11.3 = US$1.

Source: Secretaria de Salud, 2004.

PHI defines a family unit as all of the following members living in the same home: i) the head of the household; ii) the household head's spouse; iii) children younger than 18 years or single between 18 and 25 currently studying higher education; iv) children with special needs; and v) adults older than 64 years.

Perez-Vazquez et.al. (2005) found that in 2002 a total 3.8 percent of the Mexican households incurred catastrophic health expenditures, where they defined households with catastrophic expenditures as having over 30 per cent of household ability to pay. They also found differences among states in the percentage of families that faced catastrophic expenditures. The uninsured, poor, and rural household showed a higher impoverishment risk due to health expenditures. In particular, they found that 60 percent of the catastrophic expenditures were related to outpatient care and prescriptions. Catastrophic expenditure could be related to hospitalization, prescriptions, physician visits etc. Families with less financial resources could spend a good part of their income by just paying for a prescription. The most important expenditure that it is not taken into account in this study is the transportation expenditure, some families have to travel to other states or big cities in order to have a specialist check up. This implies a considerable use of household resources, and time spent in terms of labor hours. Within the most interesting results they found that in the 20 percent poorest households of the country, 36.4 percent of the catastrophic expenditures were due to prescriptions.

The objective of this paper is to present a proposal for measuring the welfare gains from PHI reform in Mexico. For pursuing this purpose, I propose to use the direct utility value gains in consumption in a macro-calibration perspective applied in a partial equilibrium approach. Moreover, for the empirical implementation of the model, I propose to use the Mexican

National Household Income Expenditure Survey (Encuestas Nacionales de Ingreso y Gasto, ENIGH) for years 2004 and 2005. These surveys provide an excellent tool for measuring the relevant variables such as income and health expenditure, and estimate the relevant measures of gains proposed.

This paper consists in five sections, including this introduction, organized as follows. In the second section I present the models proposed for analyzing the welfare gains from PHI in Mexico. The third section presents the basic characteristics of the Mexican databases which may serve for the empirical implementation of the models. The fourth section shows some welfare gains calculations using the results of the model following a calibration approach for a different risk specifications. The fifth section concludes the document.

2 Household Welfare and Risk under Censored Shocks

2.1 Public Policy Evaluation: A General Approach

The role of health as investment and consumption has been studied in detail back to the classic work by Arrow (1963), Grossman (1972), and in recent works collected by Murphy and Topel (2003.) In all of these researchs, health is a consumption good which may provide utility either directly, by increasing the utility of other consumption goods, and/or by increasing the expected life of the agents. Murphy and Topel estimations of welfare gains from health improvements for USA on the 20th century show a huge increase in the wellbeing of consumers from improving health stock, and reducing the life span of likelihood to die from several causes such as cardio problems and cancer. These results support the case for increasing research and development in the medical area given their large social returns both in short and long run.

On a different area of analysis, studies by Currie and Gruber (1996a and 1996b) have estimated the impact of welfare health reforms on household outcomes such as female labor participation and children health status. In particular Currie and Gruber (1996) analyzed how the expansion in Medicaid eligibility for pregnant women during the 1980s and 1990s affected health outcomes in the US such as child weight at born, and infant mortality. Their main finding of their studies are a huge impact of welfare expansion on decreasing infant mortality by roughly 8.5 percent among women between 15 and 44 years; while a very small significant positive effect was found in child birth weight.

In this paper I study health welfare programs in a complementary dimension to those studied in the previous papers; in particular, I analyze the role of welfare programs of public health insurance provision when serves as a mechanism which reduces the risk in consumption faced by the households. Hence, the gains of this policy implementation are in terms of the reduction of the value of risk faced under out of pocket health expenditures. The analysis of the relations between public policy and household risk for the case of unemployment insurance has been analyzed by Gruber (1997). Using the PSID data for the United States from 1968-87, and using food as proxy of consumption, Gruber finds that in the absence of unemployment insurance, consumption would fall by 22 percent, 3 times as much as it does in reality.

The model proposed in this paper follows the same line of research of Gruber (1997) but is relatively close related to the one proposed by Lucas (2003) for measuring the loses derived from business cycles, but in this case, applied to a household environment with heterogeneity in their background characteristics. In particular for this proposal, the policy gains from PHI are the value of consumption in absence of insurance needed to achieve the same utility the household have with the implementation of PHI policy.

Following Lucas, let us assume we are interested in comparing the effect of two policies, N and I. For this case, I follows from implementing the PHI reform, while N stands for the non-policy base line. Under N the household welfare is given by $U(C_N)$ where C_N is the level of consumption the household enjoys, and under the policy I this welfare is given by $U(C_I)$ with the level of consumption C_I.

Suppose that the consumer prefers I policy over N, so $U(C_I) > U(C_N)$, and let $\kappa > 0$ solve the following equality:

$$U(C_I) = U(C_N(1+\kappa)) \tag{1}$$

We call this number κ - in units of percentage of all consumption goods - *the welfare gain* of a change in policy from N to I. To evaluate the effects of policy change on many different households, we can calculate the distribution of welfare gains among them, and estimate the aggregate value of the policy by using the pair consumption-welfare gains for each household. Lucas concludes that by analyzing the policies in this way, we obtain a method that both has comprehensive units of measure and is built up from individual preferences.

The intention of this paper is measuring the value of welfare from introducing a health insurance program on those households not having any kind of protection against this type of risk. At this stage the analysis excludes those impacts of health expenditure on increasing life expectancy, and increasing health consumption as a composite good, but focuses on the gains on reducing out of pocket expenditures that reduces the household resources available for consumption.

2.2 Consumption and Risk when Health Shocks are Censored

Let us assume households utility depend upon a composite level of consumption $\tilde{C}$, and in particular, they have a constant risk aversion utility function (Pratt, 1964) of the form:

$$U(\tilde{C}) = \alpha - \beta \exp\{-\theta \tilde{C}\} \tag{2}$$

with $\alpha, \beta, \theta \in \mathbb{R}$, and $\beta, \gamma \in \mathbb{R}^{++}$ and where θ is the constant risk aversion coefficient of the household which takes values on the positive real numbers, $\theta > 0$.

Following Deaton and Muellbauer (1989) many useful results regarding the relationship between shocks under known functional forms and expected utility under constant risk aversion follows. In particular, when the lottery over an outcome $\tilde{C}$ has a known c.d.f. $F_C(\cdot)$ and utility is of the constant risk general form, then:

$$E[U(\tilde{C})] = \alpha - \beta m(-\gamma) \tag{3}$$

where $m(\cdot)$ is the moment generating function of the random variable $\tilde{C}$. Some of the implications of this result will turn to be relevant for the implementation I will suggest for the structure of health expenditure out of pocket shocks.

Let me assume the effective consumption of the household is given by the difference between its known level of income Y which represents the gross potential for consuming, and a stochastic element representing the out of pocket health expenditure $\tilde{E}$.

$$\tilde{C} = Y - \tilde{E} \tag{4}$$

Health expenditure shock $\tilde{E}$ does not provide direct utility to the households, but reduces the possibilities of the other consumption goods. Moreover, this expenditure is enforceable so the household must pay it once its value is revealed. Therefore, the household derives utility only from the net consumption after deducing the expenditure shock.

Moreover, let me also assume the expenditure shock variable $\tilde{E}$ is linked to a normal distributed latent variable $\tilde{E}^*$ with support on the real numbers with mean μ and standard deviation σ^2 which later may be household idiosyncratic. The model assuming normality in the latent variable is specified as follows:[2].

$$\begin{aligned} \tilde{E} &= \max\{0, \min\{b, \tilde{E}^*\}\} \\ E^* &\sim N(\mu, \sigma^2) \end{aligned}$$

[2] Typically, the households would have at less zero expenditure in health and in a cross section context the more they can afford is their total income.

Moreover, given the latent variable $\tilde{E}^*$ characteristics, $\tilde{E}$ is a censored normal distributed variable. Then, given $\tilde{C}$ have two components, one of them normally censored distributed, we know that $\tilde{C}$ is indeed a normal censored random variable too[3].

Additional to the net consumption linear composition defined above, I will also assume the shocks described by the variable $\tilde{E}$ are not correlated to the known and constant household's income and potential consumption given by Y. This model specification follows from the assumption that households out of pocket health expenditure are neither planned nor ex-ante known.

In absence of insurance coverage, which is the observed condition in which most of low income Mexican households live, the agents are fully facing the risk of health expenditures. Let us remind that for the particular case of a household with constant risk aversion θ utility which derives wellbeing from a normal lottery on a variable $\tilde{X}$ with mean m and standard deviation s^2 the certainty equivalent of this lottery, $CE(\tilde{X})$, is given by:

$$CE(\tilde{X}) = m - \frac{1}{2}\theta s^2 \tag{5}$$

where by definition:

$$U(CE(\tilde{X})) = EU(\tilde{X}) \tag{6}$$

Here, $CE(\tilde{X})$ shows the level of consumption under perfect certainty that leaves the household just indifferent on having the lottery $\tilde{X}$ and face uncertainty in the outcomes. Typically, the certainty equivalent of a normal lottery defined over the real space for a constant risk averse agent depends upon three elements: mean, variance, and risk aversion. Indeed, under these regular conditions $CE(\tilde{X})$: i) is monotonically increasing in mean; ii) is monotonically decreasing in the variance (risk) of the lottery or random variable; and iii) has a linear trade-off between mean and variance in terms of the coefficient of risk aversion[4].

As it was stated before, for the case of net consumption $\tilde{C}$ where households face shocks due to out of pocket expenditures $\tilde{E}$, it is natural to set these shocks taking only positive values, while there might exist a high persistence of zero shocks per period.

[3] In particular, $\tilde{C} = \max\{0, \min\{Y - E^*, Y\}\}$.

[4] By linear tradeoff I refer that every unit of additional variance, may induce the same certainty equivalent to the agent if he is compensated with $\frac{1}{2}\theta$ additional units in the lottery mean. Numerically:

$$dCE(X) = dm - \frac{1}{2}\theta ds^2 = 0$$

so, if $dCE(X) = 0$ we have:

$$\frac{dm}{ds^2}\Big|_{dCE=0} = \frac{1}{2}\theta$$

which is a linear tradeoff depending on the risk aversion of the household.

On the other hand, in absence of other wealth specifications, it is also possible to achieve a maximum level of health expenditure given by the total income the household have for that particular period. Hence, from these facts, we may have as natural censoring points $a = 0$ and $b = Y$, both having a positive mass of probability.

In this study, I will focus in a particular functional form of random variable shocks which serves as basis of analysis given the feasible estimation and the close form solution they provide: the normal censored shocks. Normal censored variables combined with a constant risk aversion agent provides us a series of close form solution to the certainty equivalence on lotteries defined over this particular family of random variables, depending of the nature of censoring involved. The main results of these two elements turns to be useful for purpose of this analysis, and for later applications involving random variables with idiosyncratic censoring and different lottery support. The summary of these results are presented in the Table 2 below while the derivation of each of them is left for the Appendix of this paper.

Table 2. Certainty Equivalent on Normal Censored Lotteries /1

$u(x) = -exp\{-\theta x\}$, $\theta < 0$, $x \in \tilde{X} \sim N(\mu, \sigma^2)$		
Variable Support	Certainty Equivalent	M-likelihood term
$\tilde{X} \in [a, \infty)$	$\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln(M_1)$	$M_1 = \left(\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})}\right)$
$\tilde{X} \in (-\infty, b]$	$\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln(M_2)$	$M_2 = \left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})}\right)$
$\tilde{X} \in [a, b]$, $a < b$	$\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln(M_3)$	$M_3 = \left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})-\Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)$

1/ See Appendix for the proof on each result.

As Table 2 shows, censoring on the support of the random variable provides an additional effect on the certainty equivalent of the lottery which indeed depends on the particular type of censoring we are modelling.

Now, if we focus on the particular case of linear shock decomposition this paper is concerned about, the assumptions already made imply that by certainty equivalent properties on linear functions, and for this case, given a lottery over $\tilde{C}$ representing the difference between a known constant Y and a random variable $\tilde{E}$:

$$CE(\tilde{C}) = Y - CE(\tilde{E})$$

Substituting the previous results for the certainty equivalence from Table 2 for a double censored variable $\tilde{E}$, and assuming constant risk aversion households, we have:

$$\begin{aligned}\tilde{C} &= Y - \tilde{E} \\ \tilde{E} &= \max\{0, \min\{Y, \tilde{E}^*\}\} \\ \tilde{E}^* &\sim N(\mu, \sigma_*^2)\end{aligned}$$

and so;

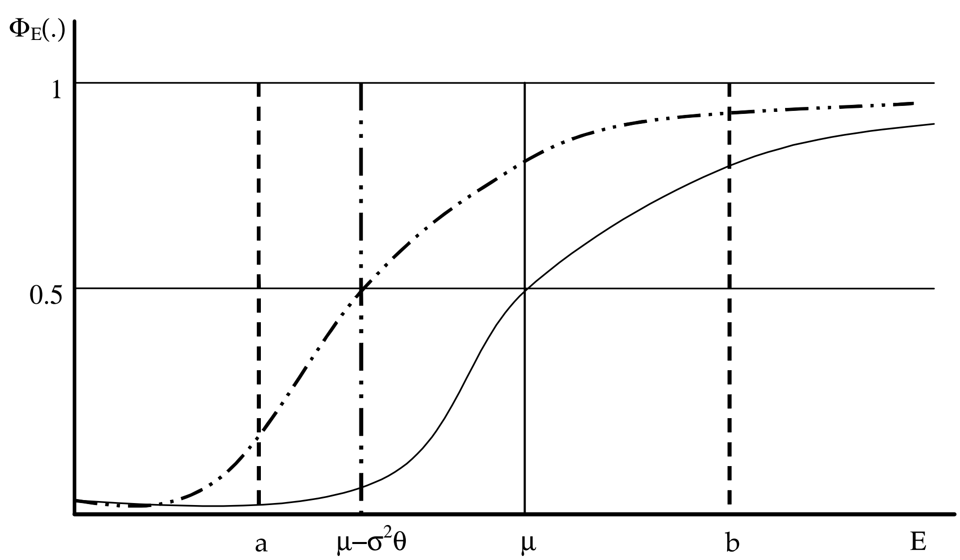
$$CE(\tilde{C}) = Y - \mu + \frac{1}{2}\sigma^2\theta + \frac{1}{\theta}\ln(M_3) \tag{7}$$

where the relative likelihood ratio M_3 is given in terms of income, risk aversion, censored values, mean, and variance by:

$$M_3 = \left(\frac{\Phi(\frac{Y-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(-\frac{(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{Y-\mu}{\sigma}) - \Phi(-\frac{\mu}{\sigma})} \right) \tag{8}$$

Further results show that likelihood ratio M_3 takes only positive values, and in particular $M_3 \in (0, \infty)$. Let us notice that as M_3 becomes smaller than 1, the term $\ln(M_1)$ turns to be more negative, reducing the value of the certainty equivalent on not having insurance. Nonetheless, the asymptotic properties under which M_3 goes to 1 would depend again on a non linear trade-off between mean and variance. These properties on the likelihood ratio play a key role in $CE(\tilde{C})$ which measures the value for the uninsured households.

Figure 2: Truncation in Shocks, Likelhood Ratio Value, and Risk Aversion (CDF Normal Analysis)



The likelihood ratio of the probabilities shows a new non-linear trade-off component of the certainty equivalent between mean and variance. This index indeed would affect the entire value of the insurance against risk, measured by $CE(\cdot)$ in a non-monotonically matter[5].

2.3 Welfare Gains and Willingness to Pay for Insurance

2.3.1 Welfare from Total Withdrawing Risk

Now, let us work on the household utility when Popular Health Insurance (PHI) is implemented. Conditional on entering, when the household takes the PHI policy needs to pay a fixed fee F units of consumption. After paying this fee, the household eliminates the uncertainty element in consumption $\tilde{E}$, so the total utility from being in the PHI is given by:

$$EU(\tilde{C}) = U(Y - F) \tag{9}$$

Let us notice that a household with PHI only cares about the fixed net consumption after paying the fee: $Y - F$.

Therefore, following Lucas's notation, the gains in welfare from being at PHI would be measured by how much we should compensate the value of consumption when risk is fully faced by the household (measured by the certainty equivalent of the lottery over the censored consumption), for making the household as well as it would be with the insurance after paying the fee. Then, by definition we have:

$$U\left((1+\kappa)(CE(\tilde{C}))\right) = U((Y - F)) \tag{10}$$

so:

$$\kappa = \frac{Y - F - CE(\tilde{C})}{CE(\tilde{C})} \tag{11}$$

and in terms of the parameters of the model:

$$\kappa = \frac{\mu - F - \frac{1}{2}\theta\sigma^2 - \frac{1}{\theta}\ln(M_3)}{Y - \mu + \frac{1}{2}\theta\sigma^2 + \frac{1}{\theta}\ln(M_3)} \tag{12}$$

This implies the welfare gains in terms of consumption are the excess of income over both the certainty equivalent of not having the insurance, and the fixed fee paid for being in the PHI, as a proportion of the non-intervention level.

[5] These results are discussed up to some detail in the appendix of this document.

2.3.2 Welfare from Changing Relative Risk

A more realistic scenario is to assume the provision of public health insurance is comparable to a government subside on the health services. This subside modifies the relative price of the health goods, and so of consumption, for those households willing to pay the fee. For a framework where health is part of a composite consumption good subject to exogenous shocks, the change in prices may be assumed to be equivalent to a change in the relative risk faced by the household.

Following this argument, let us assume that instead of totally eliminating the risk of the households due out of pocket expenditure, PHI make this variable comparable to the risk faced by one family sharing the same background characteristics but insured, and presumably with a different latent distribution. Let $\tilde{C}_0$ be the level of consumption under the current shock distribution, and $\tilde{C}_1$ be the level of consumption the household face under the public insurance policy with a different shock distribution and paying a fee F. The specification of the model then goes as follows:

$$\begin{aligned}\tilde{C}_0 &= Y - \tilde{E}_0 \\ \tilde{E}_0 &= \max\{0, \min\{Y, \tilde{E}_0^*\}\} \\ \tilde{E}_0^* &\sim N(\mu_0, \sigma_{*0}^2)\end{aligned}$$

and

$$\begin{aligned}\tilde{C}_1 &= Y - F - \tilde{E}_1 \\ \tilde{E}_1 &= \max\{0, \min\{Y, \tilde{E}_1^*\}\} \\ \tilde{E}_1^* &\sim N(\mu_1, \sigma_{*1}^2)\end{aligned}$$

In this case, the welfare gain in terms of consumption γ is given by:

$$U\left((1+\gamma)(CE(\tilde{C}_0))\right) = U(CE(\tilde{C}_1)) \tag{13}$$

And therefore:

$$\gamma = \frac{CE(\tilde{C}_1) - CE(\tilde{C}_0)}{CE(\tilde{C}_0)} \tag{14}$$

Which substituting terms from our previous results implies:

$$\gamma = \frac{(\mu_0 - \mu_1) - \frac{1}{2}\theta\left(\sigma_0^2 - \sigma_1^2\right) - \frac{1}{\theta}\left(\ln\left(M_{3,0}\right) - \ln\left(M_{3,1}\right)\right) - F}{Y - \mu_0 + \frac{1}{2}\theta\sigma_0^2 + \frac{1}{\theta}\ln\left(M_{3,0}\right)} \tag{15}$$

In this second case, the gains for the household are in terms of reduction in the mean of shocks, but also a second term depending on the variance enters. Here the trade-off between mean and variance becomes more evident than in the total risk withdrawing from the previous case. In particular, transforming the mean-variance space of health expenditures for the households totally changes the distribution of possible gains.

2.3.3 Willingness to Pay for Health Insurance

Following the same arguments above, we are able to recover the willingness to pay of a household not having insurance for the protection against the health shocks it may face. This willingness to pay is given by the maximum fee, $F^{\max}$, which make the household at least as well as it is without the insurance.

For the case of total risk withdrawing, the willingness to pay F_T of the household coincides with the following expression in terms of the value of uncertainty given by $CE(\tilde{C})$ as follows:

$$U(CE(\tilde{C})) = U\left(CE(Y - \tilde{E})\right) \leq U(Y - F_T)$$

$$F_T \leq CE(\tilde{E})$$

$$F_T^{\max} = CE(\tilde{E}) = \mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln\left(M_3\right) \tag{16}$$

Which as proportion of the total available income for consumption, then the willingness to pay is given by $\varphi^{\max}$:

$$\varphi_T^{\max} \leq \frac{\mu - \frac{1}{2}\sigma^2\theta - \frac{1}{\theta}\ln\left(M_3\right)}{Y} \tag{17}$$

Let us notice given the nature of preferences and shock structure, the willingness to pay for withdrawing the risk of health expenditure is independent of the household's income, so for two households facing the same risk, but different income, the proportional welfare gains will be higher to the lower income household.

Finally, for the case of partial risk withdrawing, i.e. equating the risk to a different level, the willingness to pay is given by:

$$U\left(CE(Y - \tilde{E}_0)\right) \leq U(CE(Y - F_P - \tilde{E}_1))$$

$$F_P \leq CE(\tilde{E}_0) - CE(\tilde{E}_1)$$

$$F_P^{\max} = CE(\tilde{E}_0) - CE(\tilde{E}_1) = (\mu_0 - \mu_1) - \frac{1}{2}\theta\left(\sigma_0^2 - \sigma_1^2\right) - \frac{1}{\theta}\left(\ln\left(M_{3,0}\right) - \ln\left(M_{3,1}\right)\right) \tag{18}$$

and as proportion of the total income available in the household we have:

$$\varphi_P^{\max} = \frac{(\mu_0 - \mu_1) - \frac{1}{2}\theta\left(\sigma_0^2 - \sigma_1^2\right) - \frac{1}{\theta}\left(\ln\left(M_{3,0}\right) - \ln\left(M_{3,1}\right)\right)}{Y} \tag{19}$$

For estimating the welfare gains of the household we need a set of idiosyncratic parameters $\Theta = \{Y, \theta, F, (\mu, \sigma^2)\}$.

The aim of this paper is to build up on this model using empirical data sets for Mexico, and recover the estimation of the set of parameters Θ for different household profiles using a Tobit parametric specification described in the following section. Nonetheless, some caveats follow from using this first approach methodology. The most important is that I am not allowing the households to save and so prepare themselves for facing a shock in health in the future; namely, the model is not permitting an optimal planning response of the household toward risk. Following this argument, the methodology proposed overestimate the gains from insurance, as households are overexposed to risk during the period analyzed.

2.3.4 Correcting Overexposure to Risk: Allowing for Portfolio Diversification

As it was already stated before, the framework proposed so far permits to estimate an upper bound for the gains of the household from having insurance on health. This subsection presents some ideas on how to improve the analysis toward a more precise measure of benefits from risk reduction of the households due to out of pocket expenditure by allowing the household the possibilities of self-insurance through precautionary insurance using some investment assets to diversify their risk.

Let us assume as before the household has an income Y which can invest at the beginning of the period in two type of assets, one which brings a gross fixed return rate r and other which brings a variable gross fixed return rate $\tilde{\imath}$. For simplicity, let us assume that the health shocks occurs at the end of the period, once the household have collected the returns of its portfolio. Finally, the household derives utility consuming the net resources available once it pays the expenses due to the out of pocket expenditures. Therefore, the timing of the model goes as follows: the households allocates its income in the assets, collects the returns, then face a health expenditure shock, and finally consumes the net resources available. Considering this timing, the household optimizes his portfolio allocation by maximizing its ex-ante expected utility which relies on his net resources available for consumption.

Let me define Z_1 to be the asset with fixed gross return r and Z_2 be the asset with gross variable return i. Then the, household allocates optimally a fraction α of its income Y to asset Z_1 and therefore $(1 - \alpha)$ to the asset Z_2. Hence, the total net resources for consumption $\tilde{W}$ is also a random variable with the following linear structure:

$$\tilde{W} = \alpha r Y + (1 - \alpha)\tilde{\imath} Y - \tilde{E}$$

And so the household optimizes his behavior by choosing α such that:

$$\max_{\{\alpha\}} E[U(\tilde{w})]$$

Where the expectation in this case is taken over the composite random variable $\tilde{W}$ which depends of two random variables: $\tilde{\imath}$ and $\tilde{E}$. Then in this case:

$$\alpha^* = \arg\max_{\{\alpha\}} E[U(\tilde{w})]$$

where α^* shows the optimal allocation of portfolio in assets of the households facing shocks. Let us notice that in this case, the expected utility is defined over a composite random variable which is the combination of the variable market return of the assets $\tilde{\imath}$ and the idiosyncratic health shocks $\tilde{E}$. In particular, even assuming the variable $\tilde{\imath}$ has a known probability distribution, its linear the combination with the censored shock on health arises a new variable which properties need to be analyzed in detail, goes beyond the purposes of this paper at this stage, and is let for a future research.

In any case, the expected results from this exercise are that the gains for the household will be different if the risk the household face each period is correlated with the volatility they face in their assets. Otherwise, the asset structure is independent of the health shocks, and the first simplifying approach proposed applies.

2.4 Idiosyncratic Distribution of Shocks: Empirical Identification

From the previous section we know the certainty equivalent and the welfare gains measures depends among other elements from the mean and variance of the latent variable distribution to the health shocks. This section presents a methodology for recovering the mean and variances of the shocks which are the parameters necessary for estimating the welfare gains and willingness to pay for insurance of the households[6].

In this case, I will assume two different approaches. In the first one, both mean and variance parameters are non functions of some relevant background characteristics but constants which in principle may be different among household cells; and the second approach these two parameters may depend on some household idiosyncratic conditions using a particular parametric specification based on the latent variable model, and particularly, in the Tobit model.

[6] Up to this point, the risk aversion coefficient will be set free to vary, and the estimation will be carried out for a set of values.

A first direct approach is to use the non-linear likelihood function to optimize in mean and variance parameters for each of the subsamples built from the original sample. Now, for the lower censored specification at a level a, an upper censoring b, and assuming the relevant sample is *i.i.d.*, we have the following log-likelihood function for the sample of size N, with observations $\{E_i, g_{1i}, ..., g_{Li}\}_{i=1}^N$, and for $\tilde{\mu}$ and $\tilde{\sigma}_*^2$ being the relevant parameters to estimate, we maximize the following log-likelihood function:

$$\begin{aligned}\log(\pounds(\tilde{\mu}, \tilde{\sigma}_*^2)) &= \sum_{i=1}^N 1(\tilde{E}_i = a) \log\left(\Phi\left(\frac{a-\tilde{\mu}}{\tilde{\sigma}_*}\right)\right) + \sum_{i=1}^N 1(\tilde{E}_i = b) \log\left(1 - \Phi\left(\frac{b-\tilde{\mu}}{\tilde{\sigma}_*}\right)\right) \\ + \sum_{i=1}^N 1(a &< \tilde{E}_i < b) \log\left(\left(\frac{1}{\tilde{\sigma}_*}\right)\frac{1}{\sqrt{2\pi\tilde{\sigma}_*^2}} \exp\left\{-\frac{1}{2}\left(\frac{\tilde{E}_i - \tilde{\mu}}{\tilde{\sigma}_*}\right)^2\right\}\right) \end{aligned} \tag{20}$$

where $1(d_i)$ is an index function that takes value 1 if d_i is true, 0 otherwise.

If on the other hand, our interest is focused in recovering the health shock expenditures $\tilde{E}$ distribution conditional on some household observables $G = (g_1, ..., g_L)$ including characteristics such as: gender of the head of the household, education of the head of the household, number of kids, age profile of kids, state, region, and other social variables. In this case, each household i is defined by a profile $(g_{1i}, ..., g_{Li})$ of these variables.

Now, let me recall the relationship between the observed health shock $\tilde{E}_i$, and the latent variable health shock $\tilde{E}_i^*$. Let me assume the health expenditure level is given by the following latent variable specification:

$$\begin{aligned}\tilde{E}_i^* &= \Gamma G_i + u_i, \text{ where } u|G \sim N(0, \sigma_*^2) \\ \tilde{E}_i &= \max\{a, \tilde{E}_i^*\}\end{aligned}$$

From here we know that $\tilde{E}^*$ is the latent variable to the observed health expenditures which satisfies the classical linear model assumptions, while $\tilde{E}$ is the lower censored observed health expenditure. Given the assumptions, we know the conditional distribution on the latent expenditure $\tilde{E}^*$ is specified by:

$$\tilde{E}^*|G \sim N(\Gamma G, \sigma_*^2)$$

where:

$$\begin{aligned}E[\tilde{E}^*|G] &= \Gamma G \\ Var[\tilde{E}^*|G] &= \sigma_*^2\end{aligned}$$

In this second case we have the following log-likelihood function for the sample of size N, with observations $\{E_i, g_{1i}, ..., g_{Li}\}_{i=1}^N$:

$$\log(\pounds(\Gamma, \sigma_*^2)) = \sum_{i=1}^N 1(\tilde{E}_i = a) \log\left(\Phi\left(\frac{a - \Gamma G_i}{\sigma_*}\right)\right) + \sum_{i=1}^N 1(\tilde{E}_i = b) \log\left(1 - \Phi\left(\frac{b - \Gamma G_i}{\sigma_*}\right)\right)$$
$$+ \sum_{i=1}^N 1(a < \tilde{E}_i < b) \log\left(\left(\frac{1}{\sigma_*}\right) \frac{1}{\sqrt{2\pi\sigma_*^2}} \exp\left\{-\frac{1}{2}\left(\frac{\tilde{E}_i - \Gamma G_i}{\sigma_*}\right)^2\right\}\right) \tag{21}$$

where as before $1(d_i)$ is an index function that takes value 1 if d_i is true, 0 otherwise.

From the MLE procedure on the previous specification we can recover the estimation on the parameters (Γ, σ_*) in the relevant sample. Nonetheless, the classical linear specification proposed above has a caveat: it assumes the covariates of mean and variance of the latent variable $\tilde{E}^*$ are orthogonal and while the mean is household determined it only provides a unique estimation of the variance independent of the covariates determining the mean.

One solution proposed to this problem is clustering the household in "cells" or sub-sets of the sample by some general profile characteristics, and then estimate the mean and variances for each of these cells. This second best approach provide variability and identification for the variance estimations and also different mean linear function. For instance, the selection of clusters may considers only basic general characteristics such as: head of the household gender, type of work (unemployed, formal, or informal), and income decile. This would give us 60 cell groups, with different variance and mean functional forms.

3 The Mexican Databases Characteristics

For estimating and testing the robustness of the implications from the model proposed in this document, we can consider several sources of information. Among these databases, the most useful for the information they provide in the micro-finance financial framework and for macro-dynamics implications is the Encuestas Nacionales de Ingreso y Gasto de los Hogares (ENIGH) collected by the Mexican National Institute of Statistics, Geography, and Informatics (INEGI), in several of their waves

The ENIGH surveys collect, periodically and systematically, socioeconomic information of the households. This information is representative at the national, rural-urban, and marginality stratum (for 2002 only, according to CONAPO's classification) levels. The main objective of this survey is to generate information on current income and expenditure structure, financial income and expenditure structure, the value of the goods and services for self-consumption, the socioeconomic characteristics of the household members, their labor conditions, and the household characteristics. For each year, the sampling process was stratified, multi-staged and by conglomerates. The final sampling unit is the household and all its members. In every stage, the selection probability is proportional to the size of the sampling unit, so the use of weighting factors is necessary to obtain the appropriate estimates.

At this stage of the paper, I used the ENIGH 2004 survey and identify the relevant income, expenditure distribution, and context variables. I built deciles by quarterly income per capita of the households, and measuring access to health insurance, by identifying all the possible sources of aggregate household health insurance and considering uninsured all those households which no member have access to health protection. From there, I also estimate the household expenditures on health by analyzing quarterly expenditures in: medicines, health care, doctor visits, and hospitals, and avoiding to include the expenditures in health prevention. Also I ignored expenditures in food as mechanism of health provision. The general distribution of access to health insurance coverage is presented below.In Figure 2 we observe

Figure 3: Distribution of Household Health Expenditure by Insurance Coverage Within Decile, Mexico 2004

	Uninsured		Insured	
Decile	**0**	**>0**	**0**	**>0**
1	37.25	54.95	3.61	4.22
2	25.95	49.16	12.20	12.74
3	22.98	46.76	11.22	18.97
4	15.90	44.08	16.36	23.70
5	16.54	37.70	17.08	28.64
6	14.62	33.64	17.70	34.04
7	18.69	28.75	16.44	36.15
8	14.00	29.17	18.34	38.48
9	11.49	27.55	15.98	45.02
10	10.85	26.66	15.83	46.60
Total	18.83	37.84	14.48	28.86
Cases	4,864,525	9,778,513	3,740,571	7,456,341

Source: ENIGH 2004, INEGI Mexico.

that considering the four lower income deciles, three in four households are uninsured and face health shocks with out of pocket expenditures, while for this same level of poverty, 1 in 2 households faced positive health expenditures without having any insurance. This is a large number compared to the proportion of household in this same level of poverty who faced positive health shocks and were insured (6 percent of the total households, i.e. 15 percent of the total low income population). Moreover, the incidence of positive health expenditures conditional on being insured is higher for those household uncovered (66 percent) to those insured (57 percent.)The next step is to study the characteristics of health shocks by gender. Previous studies suggest that the larger percentage of uninsured households where leaded by women.

Figure 4: Distribution of Household Health Expenditure by Insurance Coverage and Gender Within Decile, Mexico 2004

Decile	Uninsured				Insured			
	Men		Women		Men		Women	
	0	>0	0	>0	0	>0	0	>0
1	30.44	43.86	6.81	11.09	3.20	3.80	0.41	0.43
2	19.25	39.76	6.69	9.40	9.63	10.31	2.57	2.43
3	19.46	34.67	3.51	12.09	9.15	15.97	2.06	3.00
4	10.64	33.12	5.26	10.96	13.22	19.89	3.15	3.81
5	11.92	28.07	4.61	9.62	14.47	24.97	2.61	3.67
6	8.95	24.63	5.67	9.01	13.37	29.29	4.33	4.76
7	9.99	20.96	8.70	7.79	13.35	28.31	3.08	7.83
8	9.27	19.90	4.73	9.26	14.02	29.57	4.33	8.91
9	7.80	19.79	3.69	7.77	11.83	36.73	4.15	8.29
10	7.35	16.93	3.50	9.73	12.34	36.90	3.48	9.70
Total	13.51%	28.17%	5.32%	9.67%	11.46%	23.57%	3.02%	5.28%
Cases	3,490,265	7,279,230	1,374,260	2,499,283	2,960,945	6,091,307	779,626	1,365,034

Source: ENIGH 2004, INEGI Mexico.

Data from ENIGH 2004 reveals that for Mexico, 23 percent of all the households are leaded by women, and from there, 15 percent are uninsured. Moreover the prevalence of positive health expenditure in those households leaded by women is higher. Focusing on the lower 4 income deciles, the relative number of households uninsured and facing purely out of pocket expenditures for both men and women is larger compared to their insured peers for the same income decile.

Using this same set of data, we observe that the average of shock as percentage of income is also higher for those low income households uninsured. For instance, focusing in the first decile, households lead by a women and uninsured face as percentage of their income a quarterly shock with mean 7.6 and variance 10.5; the difference with their insured peers is enormous considering that for this same case but facing health insurance the mean reduces by half 2.9 and the variance reduces by a factor of 0.75 to 2.4.

The natural step to follow is analyze the relevance of out of pocket health expenditures for both insured and uninsured households relative to their income. In this case, Figure 3 shows that conditional on having a positive expenditure, 2004 data for low income households reveals that both uninsured households leaded by men and women consistently presents relative higher mean, and variances in their health expenditures relative to their incomes. For instance, uninsured households lead by women in first decile face an average shock that represents more than the double as percentage of their income, relative to one insured household lead by a women. Moreover, the variance of the shocks for this low income group is in particular 4 times higher for uninsured relative to insured.

Figure 5: Quarterly Income and Health Expenditure by: Income Decile, Insurance Status, and Gender

Decile	INCOME				HEALTH EXPENDITURE				HEALTH EXPENDITURE (%)			
	Uninsured		Insured		Uninsured		Insured		Uninsured		Insured	
	Men	Women	Men	Women	Men	Women	Men	Women	Men	Women	Men	Women
1	114.5	121.4	138.3	133.0	7.4	8.5	7.1	3.8	6.6%	7.6%	4.7%	2.9%
	36.5	*33.1*	*29.7*	*22.6*	*16.8*	*10.5*	*15.2*	*2.8*	*14.1%*	*10.5%*	*9.3%*	*2.4%*
2	205.3	207.7	206.4	211.2	12.6	12.8	6.9	4.0	6.2%	6.0%	3.5%	1.9%
	21.5	*21.0*	*21.5*	*18.1*	*27.8*	*16.0*	*16.8*	*4.9*	*13.6%*	*7.6%*	*9.2%*	*2.4%*
3	276.0	273.5	278.0	275.7	12.4	11.2	7.2	14.8	4.5%	4.0%	2.6%	5.6%
	20.8	*19.3*	*19.8*	*21.1*	*26.6*	*18.1*	*12.3*	*22.4*	*9.5%*	*6.2%*	*4.6%*	*8.6%*
4	348.9	349.1	349.2	353.8	16.0	14.5	13.4	7.2	4.6%	4.1%	3.8%	2.1%
	22.7	*22.1*	*21.6*	*22.2*	*31.0*	*21.2*	*27.2*	*15.8*	*8.9%*	*5.8%*	*7.6%*	*4.6%*
5	429.1	428.1	433.6	427.1	20.9	15.8	15.0	14.1	4.9%	3.7%	3.5%	3.2%
	25.3	*26.6*	*25.4*	*23.8*	*37.0*	*27.2*	*46.2*	*21.5*	*8.7%*	*6.3%*	*11.2%*	*4.8%*
6	536.8	534.1	534.0	536.8	27.1	42.4	13.7	19.0	5.0%	8.2%	2.6%	3.5%
	34.3	*35.1*	*36.3*	*32.4*	*55.8*	*92.5*	*29.5*	*48.1*	*10.0%*	*18.4%*	*5.5%*	*8.9%*
7	667.6	685.0	675.4	678.2	29.7	33.6	20.3	20.4	4.4%	4.8%	3.0%	3.0%
	43.6	*43.2*	*43.6*	*39.1*	*59.4*	*56.8*	*37.6*	*29.4*	*9.0%*	*8.0%*	*5.3%*	*4.4%*
8	889.2	898.6	890.9	904.0	45.0	42.0	28.2	30.3	5.1%	4.7%	3.2%	3.4%
	92.0	*83.7*	*85.4*	*87.2*	*90.0*	*71.8*	*76.7*	*53.4*	*10.1%*	*8.2%*	*8.7%*	*6.1%*
9	1297.4	1288.1	1302.1	1299.9	57.1	54.7	39.4	53.8	4.6%	4.2%	3.1%	4.1%
	161.4	*172.8*	*156.6*	*163.7*	*122.6*	*68.8*	*72.9*	*86.2*	*10.2%*	*5.4%*	*6.0%*	*6.4%*
10	4215.6	3686.0	3186.4	2979.2	172.5	133.7	76.6	102.9	3.4%	3.8%	2.5%	3.7%
	6062.2	*8488.7*	*2798.9*	*1538.0*	*651.1*	*298.1*	*191.2*	*261.3*	*5.3%*	*5.2%*	*5.2%*	*11.2%*
Total	668.5	815.7	1066.4	1133.8	668.5	815.7	1066.4	1133.8	5.1%	5.1%	3.0%	3.5%
	1765.2	*2877.0*	*1474.9*	*1144.0*	*1765.2*	*2877.0*	*1474.9*	*1144.0*	*10.8%*	*9.0%*	*7.2%*	*7.4%*

*/ Mean in Normal, *Standard Deviation in Italics* .

Source: ENIGH 2004, INEGI Mexico.

Heterogeneity in mean and variances of the relative health expenditure shocks between insured and uninsured households, between and within different income deciles, supports the hypothesis of heterogeneity in the welfare gains from reducing out of pocket expenditure risk.

The next section presents the estimations and results from the model proposal and divides the empirical analysis in two subsections: the first subsection discusses the results of maximum likelihood estimation of the mean and variance for the different cell-groups built considering income decile, gender of the head of the household, and insurance status; the second subsection studies the welfare gains implied by the previous estimations assuming several risk averse specifications for the agents.

4 From Model to Data: Some Calibration Results

This section presents a calibration exercise of the welfare gains using the observed behavior from the Mexican data sets described before. While this estimations are not conclusive, they provide a first approach toward what to expect from a more accurate estimation using MLE.

For the calibration exercise I considered the case of a household with a quarterly per capita income of US $200 dollars, and experimented with different specification of health shock parameter distributions and risk aversion parameters. These specifications considers pairs of mean and variance close to those observed in the previous section for those households in the low income deciles. Also, I considered the case of an insurance with full risk withdrawing as a first approach to the model for the particular case that households face a zero fee[7].

Figure 6: Welfare Gains for Different Mean-Risk Specifications, Quarterly Income US$200.

Risk Aversion Coefficient=1					
Mean	**Variance**				
	3	**15**	**24**	**42**	**72**
3	1.1%	1.1%	1.2%	1.2%	1.3%
8	3.4%	1.9%	1.8%	1.7%	1.6%
12	5.5%	3.1%	2.5%	2.1%	1.9%
20	10.2%	6.7%	5.0%	3.6%	2.8%
25	13.3%	9.6%	7.3%	5.0%	3.6%
50	32.0%	27.0%	23.5%	17.0%	10.7%
Risk Aversion Coefficient=2					
Mean	**Variance**				
	3	**15**	**24**	**42**	**72**
3	0.8%	0.7%	**	**	**
8	2.6%	1.2%	1.2%	**	**
12	4.7%	1.9%	1.5%	**	**
20	9.3%	4.0%	2.8%	**	**
25	12.4%	5.9%	4.0%	**	**
50	30.7%	21.2%	15.1%	8.8%	**
Risk Aversion Coefficient=3					
Mean	**Variance**				
	3	**15**	**24**	**42**	**72**
3	0.6%	**	**	**	**
8	2.0%	**	**	**	**
12	3.9%	**	**	**	**
20	8.4%	2.8%	**	**	**
25	11.4%	4.1%	**	**	**
50	29.4%	16.0%	10.0%	**	**

Note: " ** " Gain not defined.

The welfare gains distribution were measured as the equivalent to consumption under risk the household would like to have for being as good as it is with the new insurance policy.

[7] It is natural to believe that these measures will fall if we consider a positive fee to be paid.

Figure 7: Willingness to Pay for Different Mean-Risk Specifications, $US Dollars

Risk Aversion Coefficient=1

Mean	Variance 3	15	24	42	72
3	2.2	2.2	2.3	2.5	2.6
8	6.5	3.8	3.5	3.3	3.2
12	10.5	6.0	4.9	4.2	3.8
20	18.5	12.6	9.6	7.0	5.5
25	23.5	17.5	13.5	9.4	7.0
50	48.5	42.5	38.0	29.1	19.3

Risk Aversion Coefficient=2

Mean	Variance 3	15	24	42	72
3	1.6	1.5	**	**	**
8	5.1	2.4	2.3	**	**
12	9.0	3.6	3.0	**	**
20	17.0	7.7	5.5	**	**
25	22.0	11.2	7.8	**	**
50	47.0	35.0	26.2	16.2	**

Risk Aversion Coefficient=3

Mean	Variance 3	15	24	42	72
3	1.2	**	**	**	**
8	3.9	**	**	**	**
12	7.5	**	**	**	**
20	15.5	5.4	**	**	**
25	20.5	7.8	**	**	**
50	45.5	27.5	18.2	**	**

Note: " ** " Gain not defined.

The results of this exercise are presented in Figure 6 below. Let us notice that for a given risk aversion and mean, the higher the variance the household face, the greater the gains in consumption would be. For instance, for a health expenditure mean of 12 dollars per quarter, the gains goes from 1.9 to 5.5 percent of consumption, which is around 4 to 11 dollars per quarter. Though this measure seems relatively small, it is not once compared to the income these household have.

One interesting result is that for a mean and variance pair, the higher the risk aversion of the household, the lower the gains in term of consumption they would have. This counter-intuitive result arises from the fact that, the higher the variance of the latent variable on shocks, the higher the probability that the shock would fall on the censored values. And so for this case, the later effect seems to dominate the increase in variance spread for those positive values on shocks that lower the total willingness to pay for insurance. Intuitively: the higher the risk aversion the of the household, the less they value the insurance on a censored value lottery because the increase of the mass on the censored values seems to dominate the higher variance within the censored rank the shock take values.

The willingness to pay for the insurance for these same specifications is a second measure of welfare gains I estimated for this calibration first approach. The valuation for the insurance in this particular model is independent of the non-stochastic income of the households as a result of the constant risk aversion assumption of the households preferences. Hence, the WTP varies across mean-variance space, and the constant risk aversion of the households. As before, for a given risk aversion, the order of magnitude of valuation from insurance in terms of WTP is increasing in mean, and decreasing in variance. Figure 7 shows that all of the valuations for insurance are lower than the mean of the latent shock, particularly due to both the censored effect and the variance desutility.

These results are upper bounds for what the gains from reduction in risk would be, particularly because they assumed a total withdraw of risk and that households does not protect themselves using other portfolio options for diversifying their risk in absence of insurance. Nonetheless, the complementarity of the proposal above with other approaches such as natural experiments or a more complex dynamic general equilibrium would provide in a future research, currently in develop, a more complete evaluation of the welfare implications of this public policy for the Mexican households.

5 Conclusion

In this paper I presented a proposal for measuring the gains distribution among Mexican households from the expansion on welfare health insurance coverage. The analysis uses a structural model based on measuring the value of gains from reducing the risk on the effective consumption faced by the households.

For the case of having censored out of pocket health expenditure shocks at a lower value of zero, and a maximum upper value of the total income of the household, the certainty equivalent of a normal censored variable presents a close form solution which does not have a linear trade-off between mean and variance as in the uncensored case. Using these results, I proposed the welfare measures and the econometric procedures to recover from the data all of the relevant parameters, except the coefficient of risk aversion, which is assumed to be exogenous. These measures include two framework basis: the first assumed that insurance on the uninsured households totally withdraws the risk derived from out of pocket expenditures, while the second set of measures assumes the insurance modifies the relative risk faced by uninsured households making it comparable to those insured households sharing the same background characteristics.

Moreover, using preliminary results from the Mexican Households Survey of Income and Expenditure, I presented evidence that suggest there are large differences in the relative risk on consumption between insured and uninsured households within each decile, and across deciles. In particular, the data shows that household lead by women at the bottom of the income distribution are relatively more vulnerable to positive health out of pocket expenditure shocks, and those shocks present a relative higher mean and variance when measured as percentage of their income.

I applied the proposed methodology in a calibration procedure to recover the measures of gains in consumption equivalent, and the willingness to pay for insurance, given a particular profile of income an several specifications of risk. As we expected, welfare gains depends positively on mean, negatively on variance, and for some cases there is no gain on risk from this type of insurance, this due to the higher concentration of positive probability measure on the censored values of shocks.

To conclude, this paper is a first step into a more formal analysis of the health welfare policies implications. Further analysis implemented in a future research and linked to this work would provide a better understanding for the effects of this policy change under different model specifications considering, for instance, a natural experiment approach or a general equilibrium framework. This additional work would provide a more complete evaluation of the welfare effects of this type of public policy.

References

[1] Arrow, Kenneth. 1963. "Uncertainty and the Welfare Economics of Medical Care." *American Economic Review* LIII(5): 1-33.

[2] Basu, Anirban, David Meltzer. 2005. "Implications of spillover effects within the family for medical cost-effectiveness analysis," *Journal of Health Economics*. 24:751-773.

[3] Cutler, David and Richard J. Zeckhauser. 2000. *Handbook of Health Economics* (1):565-585.

[4] Cunha, Flavio; James Heckman, and Salvador Navarro. 2005. "Separating Uncertainty from Heterogeneity in Life Cycle Earnings." *Oxford Economic Papers,* 57: 191-261.

[5] Currie, Janet, and Jonathan Gruber. 1996. "Saving Babies: The Efficacy and Cost of Recent Expansions of Medicaid Eligibility for Pregnant Women." *Journal of Political Economy* (104):1263-1296.

[6] Currie, Janet, and Jonathan Gruber. 1996. "Health Insurance Eligibility, Utilization of Medical Care, and Child Health." *Quarterly Journal of Economics* (111):431-466.

[7] Danziger, Sheldon, Robert Haveman, and Robert Plotnick. 1981. "How Income Transfer Programs Affect Work, Savings, and the Income Distribution: A Critical Review." J*ournal of Economic Literature*, XIX:975-1028.

[8] Grossman, Michael. 1972. "On the Concept of Health and the Demand for Health." *Journal of Political Economics*, 80 (2):223-255.

[9] Gruber, Jonathan. 2000. "Cash Welfare as a Consumption Smoothing Mechanism for Divorced Mothers" *Journal of Public Economics* 75(2):157-82.

[10] Heckman, James J. and Edward Vyvlacil. 2006. "Structural Equations, Treatment Effects, and Econometric Policy Evaluation." *Econometrica*, 73(3):669-738.

[11] Heckman, James, J. 1979. "Sample Selection Bias as a Specification Error." *Econometrica* 47 (1):153-161.

[12] Knaul, Felicia; Hector Arreola Orleans; Oscar Mendez; Alejandra Martinez. 2005. *Salud Publica en Mexico* 47(1):s54-s65.

[13] Lucas, Robert E. 2003. "Macroeconomic Priorities." *American Economic Review* 93(1):1-14.

[14] Maddala, G.S. 1983. *Limited Dependent and Qualitative Variables in Econometrics.* Econometric Society Monographs No. 3. Cambridge University Press.

[15] Meyer, Bruce D. and James X. Sullivan. 2003. "Measuring the Well-Being of the Poor Using Income and Consumption," *Journal of Human Resources* 38 (Supplement):1180-1220.

[16] Meyer, Bruce D. 1995. "Natural and Quasi- Experiments in Economics," *Journal of Business & Economic Statistics* 13:151-162.

[17] Murphy, Kevin M. and Robert H. Topel. 2003. *Measuring the Gains from Medical Research: An Economic Approach.* The University of Chicago Press.

[18] ___________ . "The Value of Health and Longevity."Working Paper (in progress). The University of Chicago.

[19] Puentes Rosas, Esteban; Enrique Ruelas; Tania Martinez Monroy; Francisco Garrido Latorre. 2005. *Salud Publica en Mexico* 47(1):s12-s21.

[20] Pratt, John W. 1964. "Risk Aversion in the Small and in the Large." *Econometrica* 32 (1):122-136.

[21] Rosen, Sherwin and Derek Neal. 1999. "Theories of the Distribution of Earnings." *Chapter 7*, in *Handbook in Income Distribution.* Springler-Verlag.

[22] Secretaria de Salud. 2004. *Financiamiento Justo y Proteccion Social Universal: La Reforma Estructural del Sistema de Salud en Mexico*. Gobierno de Mexico.

[23] Secretaria de Salud. 2005. *Estrategia de Evaluacion*. Direccion General de Evaluacion del Desempeno, Subsecretaria de Innovacion y Calidad. Gobierno de Mexico.

[24] Seguro Popular de Salud. 2005. *Siete Perspectivas*. Insituto Nacional de Salud Publica.

[25] Sesma-Vazquez, Sergio; Raymundo Perez Rico; Carlos Lino Sosa Manzano; Octavio Gomez Dantes. 2005. *Salud Publica en Mexico* 47(1):s37-s46.

[26] Sesma-Vazquez, Sergio; Raymundo Perez Rico; Esteban Puentes Rosas. 2005. *Salud Publica en Mexico* 47(1):s47-s53.

[27] Stokey, Nancy L. and Robert E. Lucas, with Edward C. Prescott. 1989. *Recursive Methods in Economic Dynamics*. Harvard University Press.

[28] Willis, Robert J. and Sherwin Rosen. 1979. "Education and Self-Selection." *The Journal of Political Economics*, 87(5-2): S7-S36.

[29] World Bank. 2004. "Poverty in Mexico: An Assessment of Conditions, Trends, and Government Strategies." *World Bank: Report No. 28612-ME*. Latin America and the Caribbean Region, Poverty Reduction and Economic Management Division.

Appendix

A1. The certainty equivalent estimation for a lower censored normal variable

Claim 1: Given $U(C) = -\exp(-\theta C)$ with $\theta > 0$ being the coefficient of risk aversion, and $\tilde{C} \sim N(\mu, \sigma^2)$ where, $\mu > 0$, and $a \leq \tilde{C}$, then the certainty equivalent of $\tilde{C}$ is given by;

$$CE(\tilde{C}) = \mu - \frac{\sigma^2\theta}{2} - \frac{1}{\theta}\ln(M_1)$$

where:

$$M_1 = \left(\frac{1 - \Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1 - \Phi(\frac{a-\mu}{\sigma})} \right)$$

Proof $\square$*: By definition we know:*

$$U(CE(\tilde{C})) = E(U(\tilde{C}))$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{-\infty}^{\infty} -\exp\{-\theta\tilde{C}\} f_C(\tilde{C})dC$$

For the case of $\tilde{C}$ being a censored random variable that comes from anorther normal random variable X normal with mean μ and variance σ^2let us define the pdf associated to this censored value being:

$$f_C(y) = \frac{1}{\sigma^2\sqrt{2\pi}} \left(\frac{1}{1 - \Phi(\frac{a-\mu}{\sigma})} \right) \exp\left\{ -\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2 \right\}$$

hence we have:

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{a}^{\infty} -\exp\{-\theta\tilde{C}\} f_C(\tilde{C})dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{a}^{\infty} \left(-\exp\{-\theta\tilde{C}\}\right) \left(\frac{1}{1 - \Phi(\frac{a-\mu}{\sigma})} \right) \left(\frac{1}{\sigma^2\sqrt{2\pi}} \right) \exp\left\{ -\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2 \right\} dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{a}^{\infty} -\exp\{-\theta\tilde{C}\} \frac{1}{\sigma^2\sqrt{2\pi}} \left(\frac{1}{1 - \Phi(\frac{a-\mu}{\sigma})} \right) \exp\left\{ -\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2 \right\} dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)\int_a^\infty \exp\{-\theta\tilde{C}\}\exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2\right\}dC$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)* \\ &\int_a^\infty \exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\theta\tilde{C}\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)* \\ &\int_a^\infty \exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\theta\tilde{C}+\mu\theta-\mu\theta+\frac{\sigma^2\theta^2}{2}-\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)* \\ &\int_a^\infty \exp\left\{-\left(\theta C+\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\mu\theta+\frac{\sigma^2\theta^2}{2}\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)* \\ &\int_a^\infty \exp\left\{-\frac{1}{2}\left(\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2+2\theta(\tilde{C}-\mu)+\frac{\sigma^2\theta^2}{2}\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)* \\ &\int_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)^2+2\theta\sigma^2(\tilde{C}-\mu)+\sigma^4\theta^2\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right) * \int_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)+\sigma^2\theta\right)^2-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right) * \int_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)+\sigma^2\theta\right)^2-\left(\theta\mu-\frac{\sigma^2\theta^2}{2}\right)\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right) * \int_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\} * \int_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right) * \frac{1}{\sigma^2\sqrt{2\pi}}\int_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right) * \frac{1}{\sigma^2\sqrt{2\pi}}\int_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-(\mu-\sigma^2\theta)\right)^2\right\}dC$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right) * \\
& \quad \frac{1}{\sigma^2\sqrt{2\pi}}\int\limits_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-(\mu-\sigma^2\theta)\right)^2\right\} dC
\end{aligned}
$$

Let us define a variable J which is a normal distribution with mean "$\mu - \sigma^2\theta$" and variance "σ^2.", in particular:

$$f_J(j) = \frac{1}{\sigma^2\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}\left(j-(\mu-\sigma^2\theta)\right)^2\right\}$$

then by J being such a normal with different mean we know:

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)\frac{1}{\sigma^2\sqrt{2\pi}}\int\limits_a^\infty \exp\left\{-\frac{1}{2\sigma^2}\left(j-(\mu-\sigma^2\theta)\right)^2\right\} dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{1-\Phi(\frac{a-\mu}{\sigma})}\right)\left(1-\Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})\right)$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})}\right)$$

$$\exp\{-\theta CE(\tilde{C})\} = \exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})}\right)$$

$$-\theta CE(\tilde{C}) = -\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right) + \ln\left(\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})}\right)$$

$$\theta CE(\tilde{C}) = \mu\theta - \frac{\sigma^2\theta^2}{2} - \ln\left(\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})}\right)$$

$$CE(\tilde{C}) = \mu - \frac{1}{2}\theta\sigma^2 - \frac{1}{\theta}\ln\left(\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})}\right) \qquad \blacksquare$$

A2. The certainty equivalent estimation for a upper censored normal variable

Claim 2: Given $U(C) = -\exp(-\theta C)$ with $\theta > 0$ being the coefficient of risk aversion, and $\tilde{C} \sim N(\mu, \sigma^2)$ where, $\mu > 0$, and $\tilde{C} \leq b$, then the certainty equivalent of $\tilde{C}$ is given by;

$$CE(\tilde{C}) = \mu - \frac{\sigma^2\theta}{2} - \frac{1}{\theta}\ln(M_2)$$

where

$$M_2 = \left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})} \right)$$

Proof $\square$*: By definition we know:*

$$U(CE(\tilde{C})) = E(U(\tilde{C}))$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{-\infty}^{\infty} -\exp\{-\theta\tilde{C}\} f_C(\tilde{C})dC$$

For the case of $\tilde{C}$ being a censored random variable that comes from anorther normal random variable X normal with mean μ and variance σ^2let us define the pdf associated to this censored value being:

$$f_C(y) = \frac{1}{\sigma^2\sqrt{2\pi}} \left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})} \right) \exp\left\{ -\frac{1}{2}\left(\frac{y-\mu}{\sigma} \right)^2 \right\}$$

hence we have:

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{-\infty}^{b} -\exp\{-\theta\tilde{C}\} f_C(\tilde{C})dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{-\infty}^{b} \left(-\exp\{-\theta\tilde{C}\} \right) \left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})} \right) \left(\frac{1}{\sigma^2\sqrt{2\pi}} \right) \exp\left\{ -\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma} \right)^2 \right\} dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{-\infty}^{b} -\exp\{-\theta\tilde{C}\} \frac{1}{\sigma^2\sqrt{2\pi}} \left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})} \right) \exp\left\{ -\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma} \right)^2 \right\} dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)\int\limits_{-\infty}^{b}\exp\{-\theta\tilde{C}\}\exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\theta\tilde{C}\right\}dC$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\theta\tilde{C}+\mu\theta-\mu\theta+\frac{\sigma^2\theta^2}{2}-\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\int\limits_{-\infty}^{b}\exp\left\{-\left(\theta C+\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\mu\theta+\frac{\sigma^2\theta^2}{2}\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2}\left(\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2+2\theta(\tilde{C}-\mu)+\frac{\sigma^2\theta^2}{2}\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)^2+2\theta\sigma^2(\tilde{C}-\mu)+\sigma^4\theta^2\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC\end{aligned}$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)+\sigma^2\theta\right)^2-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)+\sigma^2\theta\right)^2-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}dC$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= \left(\frac{-1}{\Phi(\frac{b-\mu}{\sigma})}\right)\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}* \\ &\frac{1}{\sigma^2\sqrt{2\pi}}\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= -\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\frac{1}{\sigma^2\sqrt{2\pi}}\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= -\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\frac{1}{\sigma^2\sqrt{2\pi}}\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-(\mu-\sigma^2\theta)\right)^2\right\}dC\end{aligned}$$

$$\begin{aligned}-\exp\{-\theta CE(\tilde{C})\} &= -\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})}\right)* \\ &\frac{1}{\sigma^2\sqrt{2\pi}}\int\limits_{-\infty}^{b}\exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-(\mu-\sigma^2\theta)\right)^2\right\}dC\end{aligned}$$

Again, let us define a variable J which is a normal distribution with mean "$\mu - \sigma^2\theta$" and variance "σ^2.", in particular:

$$f_J(j) = \frac{1}{\sigma^2\sqrt{2\pi}} \exp\left\{ -\frac{1}{2\sigma^2} \left(j - (\mu - \sigma^2\theta) \right)^2 \right\}$$

then, following the same argument as in Claim 1:

$$\begin{aligned} -\exp\{-\theta CE(\tilde{C})\} \quad &= \quad -\exp\left\{ -\left(\mu\theta - \frac{\sigma^2\theta^2}{2} \right) \right\} \left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})} \right) \frac{1}{\sigma^2\sqrt{2\pi}} * \\ & \int_{-\infty}^{b} \exp\left\{ -\frac{1}{2\sigma^2} \left(j - (\mu - \sigma^2\theta) \right)^2 \right\} dC \end{aligned}$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{ -\left(\mu\theta - \frac{\sigma^2\theta^2}{2} \right) \right\} \left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})} \right) \left(\Phi(\frac{b - (\mu - \sigma^2\theta)}{\sigma}) \right)$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{ -\left(\mu\theta - \frac{\sigma^2\theta^2}{2} \right) \right\} \left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})} \right)$$

$$\exp\{-\theta CE(\tilde{C})\} = \exp\left\{ -\left(\mu\theta - \frac{\sigma^2\theta^2}{2} \right) \right\} \left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})} \right)$$

$$-\theta CE(\tilde{C}) = -\left(\mu\theta - \frac{\sigma^2\theta^2}{2} \right) + \ln\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})} \right)$$

$$\theta CE(\tilde{C}) = \mu\theta - \frac{\sigma^2\theta^2}{2} - \ln\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})} \right)$$

$$CE(\tilde{C}) = \mu - \frac{1}{2}\theta\sigma^2 - \frac{1}{\theta}\ln\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})} \right) \quad \blacksquare$$

A3. The certainty equivalent estimation for a double censored normal variable

Claim 3: Given $U(C) = -\exp(-\theta C)$ with $\theta > 0$ being the coefficient of risk aversion, and $\tilde{C} \sim N(\mu, \sigma^2)$ where, $\mu > 0$, and $a \leq \tilde{C} \leq b$, then the certainty equivalent of $\tilde{C}$ is given by;

$$CE(\tilde{C}) = \mu - \frac{\sigma^2\theta}{2} - \frac{1}{\theta}\ln(M_3)$$

where

$$M_3 = \frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}$$

Proof □*: By definition we know:*

$$U(CE(\tilde{C})) = E(U(\tilde{C}))$$

$$-\exp\{\theta CE(\tilde{C})\} = \int_{-\infty}^{\infty} -\exp\{\theta\tilde{C}\} f_C(\tilde{C})dC$$

For the case of $\tilde{C}$ being a censored random variable that comes from anorther normal random variable X normal with mean μ and variance σ^2let us define the pdf associated to this double censored value being:

$$f_C(y) = \frac{1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right)\exp\left\{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right\}$$

hence we have:

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{a}^{b} -\exp\{-\theta\tilde{C}\} f_C(\tilde{C})dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{a}^{b}\left(-\exp\{-\theta\tilde{C}\}\right)\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right)\left(\frac{1}{\sigma^2\sqrt{2\pi}}\right)\exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \int_{a}^{b} -\exp\{-\theta\tilde{C}\}\frac{1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right)\exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2\right\}dC$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \\
&\int_a^b \exp\{-\theta\tilde{C}\}\exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2\right\}dC
\end{aligned}
$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \\
&\int_a^b \exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\theta\tilde{C}\right\}dC
\end{aligned}
$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \\
&\int_a^b \exp\left\{-\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\theta\tilde{C}+\mu\theta-\mu\theta+\frac{\sigma^2\theta^2}{2}-\frac{\sigma^2\theta^2}{2}\right\}dC
\end{aligned}
$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \\
&\int_a^b \exp\left\{-\left(\theta C+\frac{1}{2}\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2-\mu\theta+\frac{\sigma^2\theta^2}{2}\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC
\end{aligned}
$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \\
&\int_a^b \exp\left\{-\frac{1}{2}\left(\left(\frac{\tilde{C}-\mu}{\sigma}\right)^2+2\theta(\tilde{C}-\mu)+\frac{\sigma^2\theta^2}{2}\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC
\end{aligned}
$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} &= \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \\
&\int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)^2+2\theta\sigma^2(\tilde{C}-\mu)+\sigma^4\theta^2\right)-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC
\end{aligned}
$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)+\sigma^2\theta\right)^2-\mu\theta+\frac{\sigma^2\theta^2}{2}\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\left(\tilde{C}-\mu\right)+\sigma^2\theta\right)^2-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = \frac{-1}{\sigma^2\sqrt{2\pi}}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \frac{1}{\sigma^2\sqrt{2\pi}}\int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\}dC$$

$$-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta-\frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)* \frac{1}{\sigma^2\sqrt{2\pi}}\int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-(\mu-\sigma^2\theta)\right)^2\right\}dC$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right) * \\
&\quad \frac{1}{\sigma^2\sqrt{2\pi}}\int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C} - (\mu - \sigma^2\theta)\right)^2\right\} dC
\end{aligned}
$$

Again, let us define a variable J which is a normal distribution with mean "$\mu - \sigma^2\theta$" and variance "σ^2.", in particular:

$$
f_J(j) = \frac{1}{\sigma^2\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma^2}\left(j - (\mu - \sigma^2\theta)\right)^2\right\}
$$

then:

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right) * \\
&\quad \frac{1}{\sigma^2\sqrt{2\pi}}\int_a^b \exp\left\{-\frac{1}{2\sigma^2}\left(j - (\mu - \sigma^2\theta)\right)^2\right\} dC
\end{aligned}
$$

$$
\begin{aligned}
-\exp\{-\theta CE(\tilde{C})\} \quad &= \quad -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{1}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right) * \\
&\quad \left(\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})\right)
\end{aligned}
$$

$$
-\exp\{-\theta CE(\tilde{C})\} = -\exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right)
$$

$$
\exp\{-\theta CE(\tilde{C})\} = \exp\left\{-\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right)\right\}\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right)
$$

$$
-\theta CE(\tilde{C}) = -\left(\mu\theta - \frac{\sigma^2\theta^2}{2}\right) + \ln\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right)
$$

$$\theta CE(\tilde{C}) = \mu\theta - \frac{\sigma^2\theta^2}{2} - \ln\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right)$$

$$CE(\tilde{C}) = \mu - \frac{1}{2}\theta\sigma^2 - \frac{1}{\theta}\ln\left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})}\right) \quad \blacksquare$$

A4. Other Properties of the "M" Likelihood Ratio with Censored Variables

The ceteris-paribus effects over $CE(X)$ are presented below.

Table A.4.1. M Likelihood Ratio, Partial Derivatives/1

Derivative	$M_1 = \left(\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})}\right)$	$M_2 = \left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})}\right)$	$M_3 = \left(\frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma})-\Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})}\right)$
$\frac{\partial}{\partial\mu}M(\cdot)$	>0	>0	$\nabla^*/$
$\frac{\partial}{\partial\sigma}M(\cdot)$	?	?	$\nabla^*/$
$\frac{\partial}{\partial\theta}M(\cdot)$	<0	>0	$\nabla^*/$
$\frac{\partial}{\partial a}M(\cdot)$	<0	$N.A.$	$\nabla^*/$
$\frac{\partial}{\partial b}M(\cdot)$	$N.A.$	<0	$\nabla^*/$

$N.A.$ Not applies for this case.

$\nabla^*/$ The sign would depend on a and b relative position with respect to μ.

1/ See Appendix for the proof on each result.

Table A.4.2. Certainty Equivalent on censored Lotteries, Derivatives /2

$u(x) = -exp\{-\theta x\},\ x \in \tilde{X} \sim N(\mu, \sigma^2)$

Variable Support	$\tilde{X} \in [a, \infty)$	$\tilde{X} \in (-\infty, b]$	$\tilde{X} \in [a, b]$
$\frac{\partial}{\partial\mu}CE(\tilde{X})$	$1-\frac{1}{\theta}(\frac{\partial M_1}{\partial\mu})^{-1}$	$1-\frac{1}{\theta}(\frac{\partial M_2}{\partial\mu})^{-1}$	$1-\frac{1}{\theta}(\frac{\partial M_3}{\partial\mu})^{-1}$
$\frac{\partial}{\partial\sigma}CE(\tilde{X})$	$(-1)(\frac{1}{2}\theta + (\frac{\partial M_1}{\partial\sigma})^{-1})$	$(-1)(\frac{1}{2}\theta + (\frac{\partial M_2}{\partial\sigma})^{-1})$	$(-1)(\frac{1}{2}\theta + (\frac{\partial M_3}{\partial\sigma})^{-1})$
$\frac{\partial}{\partial\theta}CE(\tilde{X})$	$\frac{1}{\theta^2}\ln(M_1) - \frac{1}{2}\sigma^2 - \frac{1}{\theta}\frac{1}{M_1}\frac{\partial M_1}{\partial\theta}$	$\frac{1}{\theta^2}\ln(M_2) - \frac{1}{2}\sigma^2 - \frac{1}{M_1}\frac{\partial M_1}{\partial\theta}\frac{1}{\theta}$	$\frac{1}{\theta^2}\ln(M_3) - \frac{1}{2}\sigma^2 - \frac{1}{M_1}\frac{\partial M_1}{\partial\theta}\frac{1}{\theta}$
$\frac{\partial}{\partial a}CE(\tilde{X})$	$-\frac{1}{\theta}\frac{1}{M_1}\frac{\partial M_1}{\partial a}$	$N.A.$	$-\frac{1}{\theta}\frac{1}{M_3}\frac{\partial M_3}{\partial a}$
$\frac{\partial}{\partial b}CE(\tilde{X})$	$N.A.$	$-\frac{1}{\theta}\frac{1}{M_2}\frac{\partial M_2}{\partial b}$	$-\frac{1}{\theta}\frac{1}{M_3}\frac{\partial M_3}{\partial b}$

2/ See Appendix for the proof on each result.

This section explores other properties of the likelohood ratio element of the certainty equivalent, "M", for each of the three possible censoring scenarios: lower censoring, upper censoring, and double censoring. In principle, this ratio is always positive, nontheless, the quick convergence to 1 and zero have strong impacts on the certainty equivalent of the lotteries, depending on the censor parameter, the risk aversion, and the relative position in the (mean-variance) space.

Result A1. The additional term of the $CE(\tilde{E})$ of a lower normal censored variable with a constant risk aversion utility is less or equal than 1 iff:

$$\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})} \leq 1$$

Figure 8: M-Likelihood Ratio Function, Mean-Variance Space, $\theta = 2$

Figure A) Lower truncation case; a=0.

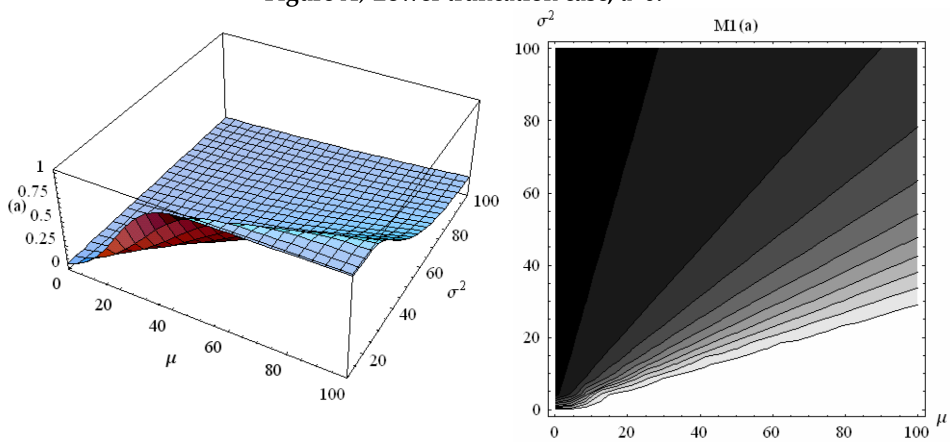


Figure B) Upper truncation case; b=1000.

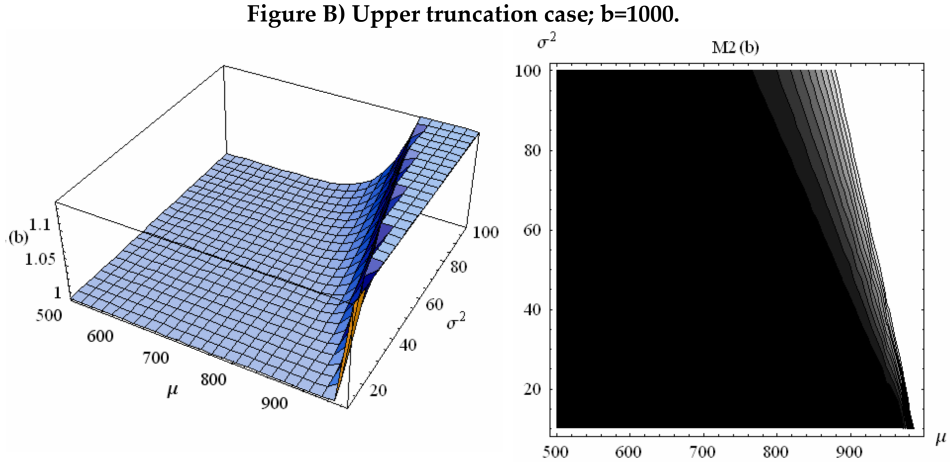
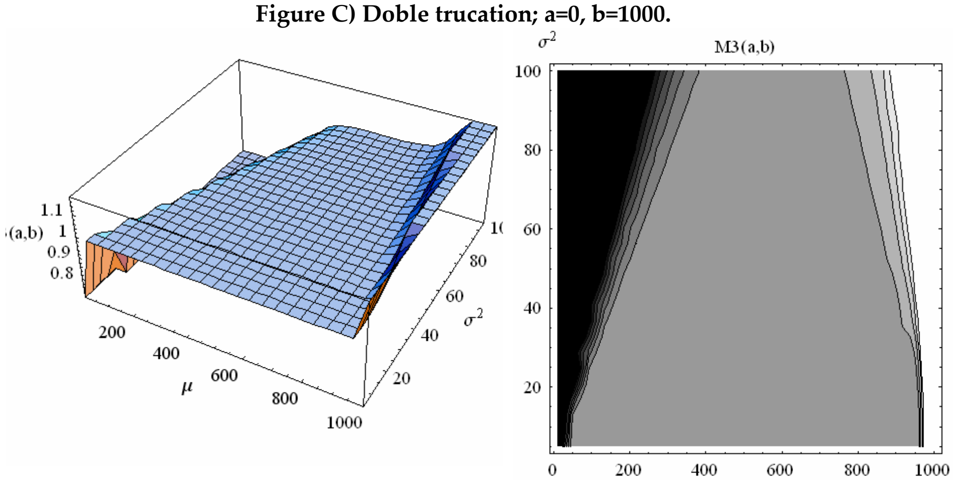


Figure C) Doble trucation; a=0, b=1000.



$$\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right) \leq \Phi(\frac{a-\mu}{\sigma})$$

$$\frac{a-(\mu-\sigma^2\theta)}{\sigma} \leq \frac{a-\mu}{\sigma}$$

$$-\mu+\sigma^2\theta \leq -\mu$$

$$\sigma^2\theta \leq 0$$

The additional term of the $CE(\tilde{E})$ of a normal censored variable with a constant risk aversion utility is positive iff:

$$M_1 = \frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})} \geq 0$$

$$1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right) \geq 0$$

$$1 \geq \Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)$$

Then, as long as $a \in \mathbb{R}$ we have: $\frac{1-\Phi\left(\frac{a-(\mu-\sigma^2\theta)}{\sigma}\right)}{1-\Phi(\frac{a-\mu}{\sigma})} > 1$ for a given μ,θ, and, σ^2.

Result A2. The additional term of the $CE(\tilde{E})$ of a normal censored variable with a constant risk aversion utility is less or equal than 1 iff:

$$M_2 = \frac{\Phi\left(\frac{b-(\mu-\sigma^2\theta)}{\sigma}\right)}{\Phi(\frac{b-\mu}{\sigma})} \leq 1$$

$$\left(\frac{b-(\mu-\sigma^2\theta)}{\sigma}\right) \leq (\frac{b-\mu}{\sigma})$$

$$b-(\mu-\sigma^2\theta) \leq b-\mu$$

$$-\mu+\sigma^2\theta \leq -\mu$$

$$\sigma^2\theta \leq 0$$

he additional term of the $CE(\tilde{E})$ of a normal censored variable with a constant risk aversion utility is positive iff:

$$\frac{\Phi\left(\frac{b-(\mu-\sigma^2\theta)}{\sigma}\right)}{\Phi(\frac{b-\mu}{\sigma})} \geq 0$$

$$\Phi\left(\frac{b-(\mu-\sigma^2\theta)}{\sigma}\right) \geq 0$$

Then, as long as $b \in \mathbb{R}$ we have: $\frac{\Phi\left(\frac{b-(\mu-\sigma^2\theta)}{\sigma}\right)}{\Phi(\frac{b-\mu}{\sigma})} > 1$ for a given μ,θ, and, σ^2.

Result A3. The additional term of the $CE(\tilde{E})$ of a double normal censored variable with a constant risk aversion utility is less or equal than 1 iff:

$$M_3 = \frac{\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma})}{\Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})} \geq 1$$

$$\Phi(\frac{b-(\mu-\sigma^2\theta)}{\sigma}) - \Phi(\frac{a-(\mu-\sigma^2\theta)}{\sigma}) \geq \Phi(\frac{b-\mu}{\sigma}) - \Phi(\frac{a-\mu}{\sigma})$$

$$\Phi(\frac{b-\mu+\sigma^2\theta)}{\sigma}) - \Phi(\frac{b-\mu}{\sigma}) \geq \Phi(\frac{a-\mu+\sigma^2\theta)}{\sigma} - \Phi(\frac{a-\mu}{\sigma})$$

$$\int_{b}^{b+\sigma^2\theta} \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\} dC \geq \int_{a}^{a+\sigma^2\theta} \exp\left\{-\frac{1}{2\sigma^2}\left(\tilde{C}-\mu+\sigma^2\theta\right)^2\right\} dC$$

This inequality will depend upon how the interval $[a, b]$ behaves with respect to μ and no general result holds. On the other hand, as long as $a, b \in \mathbb{R}$ and $a < b$, by construction the M_3 term is positive.

Result A4. The additional term of the $CE(\tilde{E})$ of a normal censored variable with a constant risk aversion utility dissapears if $\theta \to 0$, i.e. as the consumer turns to be risk neutral. If this is the case, then the certainty equivalent of the lottery turns to be the mean of the un-censored distribution.