An approach to the non-normal behavior of hedge fund indices using Johnson distributions.

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Abstract- In this paper we consider Johnson distributions as a tool to analyze and model the non normal behavior of hedge fund indices. After effectively obtaining the parameters for these distributions in the case of the CSFB /Tremont indices, we use the Omega parameter to test how efficiently the Johnson distributions reflect the presence of occasional large negative returns. We conclude that, although Johnson distributions yield improved models and confirm that skewness and kurtosis are relevant in the evaluation of performance, they still fail to capture a substantial part of the tail risk, specially in those cases when we have high kurtosis.

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1. Introduction.

It is a well accepted empirical fact that returns, whether from long only stocks or bonds or hedge funds, often show greater tendency to produce extreme returns than a normal distribution. But in the case of hedge funds or portfolios of hedge funds, in addition to the presence of these "fat tails" the departure from normality often includes asymmetry and the so called "short-option" behavior, in which occasional large negative returns appear scattered among many small positive returns.

This departure from normality is partly a consequence of the intrinsic design of hedge fund strategies, for which the main objective is to generate positive returns independent from market conditions. While long-only managers define their returns relative to a benchmark and hence they obtain a fairly normal distribution, hedge fund managers define their return objective in absolute terms, not relative to market or benchmark, striving to avoid any market correlation. Due to the legal status of hedge funds, their managers have the possibility to use derivatives, short sell and explicit leverage to raise returns or cushion risk. As a consequence of these leveraged positions, the distributions of returns frequently show occasional losses which under the normality hypothesis would be highly improbable. There are also other factors which are related to the non-normality observed in hedge fund returns: the paucity of data, the lack of statistical rigour in some of the information available, the presence of autocorrelation, etc.

Since hedge funds depart strongly from the assumption of (log)normally distributed returns without serial correlation, performance measures such as the Sharpe ratio or traditional approaches through mean-variance analysis seem inadequate, since they ignore the effect of higher moments. This had led to the introduction of new methods to measure risk and performance for non-normal distributions. We can recall some recent examples in this direction:

- The Omega measure of Keating and Shadwick (2002) which provides a new approach to study return distributions and may be used as a performance measure. The Omega function captures higher moment information and also incorporates sensitivity to return levels.

- Modified Value at Risk (Signer and Favre, 2002), a risk measure in which the usual VaR obtained from a normal distribution is adjusted for the estimated skewness and kurtosis using the Cornish-Fisher expansion.

- Adjusting Sharpe ratio to incorporate information from downside deviation (Johnson, D., Macleod, N. and C.Thomas, 2002).

From an unconditional point of view, one way to deal with non normality is to try to find some other known distribution which fits the empirical distribution of returns. Several attempts have been done in this direction, for example using Student's t distributions or Pareto-like distributions. Having an explicit model which takes into consideration the skewness and kurtosis of returns gives the possibility of performing simulations of the behavior of returns, not only to analyze future scenarios, or to obtain measures of risk and performance, but also to replicate or complete faulty sets of data.

In this paper we will consider Johnson (1949) system of S_U and S_B distributions as a system with which we can consistently approximate the empirical distributions of hedge fund returns. Johnson S_U distributions have already been mentioned in some attempts to approximate the non-normal behavior of stock returns, [1] but there is little information on the numerical efficiency of these models when applied to actual market data or on their power to capture the effects of infrequent but largely negative returns which characterize the distributions of some hedge fund strategies.

Hence, using a fund index as proxy, we will examine the efficiency of Johnson system to model the distributions of different hedge fund strategies, in particular those who show a "short option" behavior. Although the modelled distributions reflect the presence of substantial tail risk, the large negative returns are often too rare to affect the distribution parameters and may remain undetected. To see if this is the case, we will use the Omega performance measure of Keating and Shadwick (2002) as a test of how efficiently the Johnson distributions reflect this "short option" effect.

2. Hedge Fund Indexes.

When studying hedge fund performance from a statistical point of view we face some difficulties. First, there is the pausity of data: due to the short history of hedge funds and the fact that the hedge fund industry is largely unregulated and it is not mandatory for funds to report performance, we often find ourselves with not enough data available. Moreover,

[1] For example, from a conditional point of view, Johnson's S_U-distributions have been used to propose GARCH-S_U models for the estimation of extreme tail behavior of stock market indices (Choi, 2001).

until very recently, there was a total lack of transparency of hedge fund investment strategy and statistical issues concerning the rigour of hedge fund performance data were not discussed at all. This situation has begun to change and some hedge fund managers now present their results and their allocation strategies in a public or semipublic way. Since obtaining direct and valid information from hedge fund managers about the performance or composition of hedge funds is difficult for a non-investor, and since managers follow different strategies which is not possible to compare, we will not study the behavior of individual hedge funds and instead we will use hedge fund indices as proxy. These indices are provided by several data vendors which set up hedge fund data bases and they are meant to be used as benchmarks for measuring hedge fund performance. Each data vendor provides a global index as well as subindices which correspond to different investment strategies. There are at present at least seven major databases available in the market, each one with its own policy for selecting and evaluating the hedge funds in the index. Thus, although these vendors usually provide accurate information, there are limitations for using and comparing these indices:

1. There can be different biases in the database: survivor bias (exclusion of defunct funds), autocorrelation bias and selection bias. The limitation of the sample size and the inaccuracy of data can also distort results.

2. Each data vendor has a different policy for excluding/including funds in the index, classify investment strategies or determining the weighting of funds in the index. Hence it is difficult to compare indices from different vendors. In fact, there can be low correlation between different indices representing the same investment strategy.

3. There is a wide variation in sub-index categorization, so the comparison across global indices is unrealistic.

In view of the above, it seems that hedge fund indices are still not a true benchmark for performance. However, we can use them as a proxy of the average behavior associated with a specific strategy.

In this paper I will consider in particular the CSFB/Tremont hedge fund indices, with data covering from 1994 to 2003. This index is constructed with TASS database, which tracks about 2300 funds. It is composed of a global index together with 9 subindices provided using a subset of 650 funds, each subindex corresponding to a different strategy: Convertible Arbitrage, Dedicated Short Bias, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Global Macro, Long/Short and Managed Futures. Funds are reselected on a quarterly basis and the selection criteria include a minimum fund size of $ 10MM and the requirement that the fund should have an audited financial statement. In the following table we have a summary of the basic statistics for the CSFB indices. For comparison, we have included at the bottom statistics for the S&P 500 and for the JP Morgan treasury bond index.

Table 1: Basic statistics for CSFB/Tremont hedge fund indices*

	Annualized returns	Annualized volatility	Skewness	Kurtosis	Jarque-Bera
CSFB/Tremont HF Index	10.59%	8.86%	0.107	4.27	7.57
Convertible Arbitrage	10.14%	4.90%	-1.56	6.77	109.01
Ded Short Bias	2.11%	18.34%	0.839	4.85	28.41
Emerging Markets	6.46%	18.49%	-0.473	6.04	46.21
Equity Mkt Ntrl	10.45%	3.20%	0.139	2.959	0.35
Event Driven	10.37%	6.27%	-3.279	**23.225**	2053
Fixed Inc Arb	6.64%	4.09%	-3.168	18.05	1211.04
Global Macro	14.15%	12.65%	-0.025	4.495	10.16
Long/Short	11.56%	11.47%	0.245	5.7631	35.75
Managed Futures	7.46%	12.08%	0.034	3.663	2.01
S&P	6.76%	15.66%	-0.36	2.85	2.61
JPMTUS	1.46%	4.28%	-0.193	3.05	0.24

*Evaluated over a sample of 109 monthly returns from 1/1994 to 2/2003.

When Jarque-Bera test for normality yields a value > 5.99, we must reject the normality hypothesis at a 5% significance level. Hence, with the exception of Equity Market Neutral and Managed Futures (together with treasury bonds and S & P500) in all cases we have to reject the normality hypotheses. In general all indices have negative skewness and high kurtosis, which means that large negative returns are more likely than would be the case under a normal distribution. This is specially the case for the Event Driven and Fixed Income Arbitrage indices. It is also interesting to note that, while in long-only equity investments the volatility usually is around 15% or 20% per year, average volatility for hedge funds is significantly lower. Finally, on a risk adjusted basis (mean return/standard deviation) the index that ranks higher is Equity Market Neutral.

As an example of their departure from normality, figures (1) and (2) compare the frequency of returns of Event Driven and Fixed Income Arbitrage indices with the normal curve with the same mean and standard deviation. In both cases we remark the presence of isolated large negative returns.

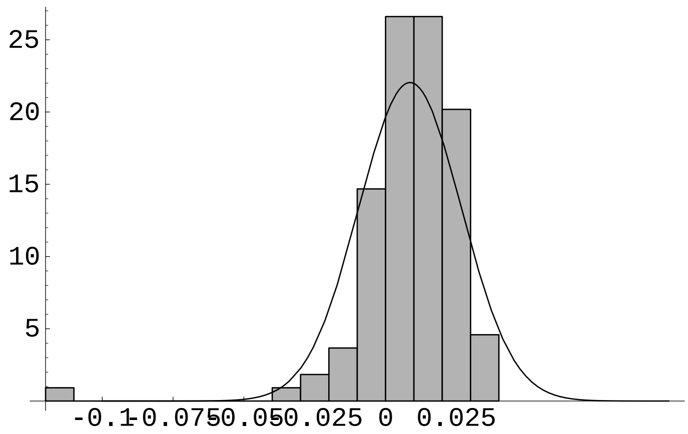

Figure 1: Event Driven Index

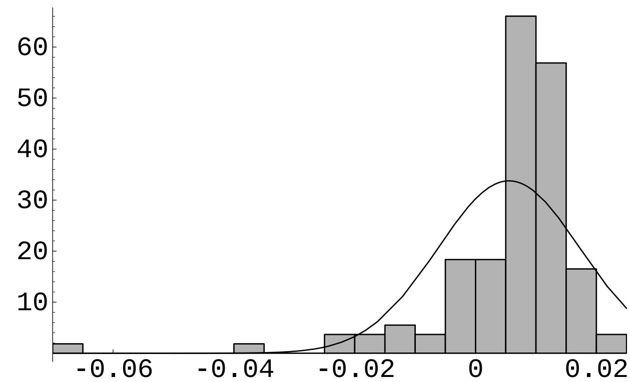

Figure 2: Fixed Income Arbitrage

It seems reasonable to think that a hedge fund manager must try to avoid symmetry of returns by deforming the distribution to the right (more profit and fewer losses). Surprisingly, in most of these indices things appear to go the other way around: the distribution gains in negative skewness and gets higher kurtosis. It seems that the fund manager improves the mean return by shortening the right hand tail and fattening the left one. The left tails reflect high leverage, which correspond to a payoff similar to strategies that have short put option-like exposures. It seems the improved mean of hedge funds returns is just the premium the market is paying for the exposure to leptokurtic distribution of returns.

But leverage is not the only possible reason for the departure from normality. There is also serial correlation. In the next table we present the autocorrelation coefficients of each of the indices, considering lags from 1 to 5. Each correlation coefficient can be considered as statistically significant at the 5% level if it lies outside ± 0.22 and significant at the 1% level if it lies outside ± 0.3.

Table 2: Autocorrelation of hedge fund indices

	AC(1)	AC(2)	AC(3)	AC(4)	AC(5)
CSFB Tremont	0.112	0.041	-0.005	-0.08	0.053
Convertible Arbitrage	0.562	0.427	0.154	0.127	0.08
Ded Short Bias	0.067	-0.073	-0.035	-0.103	-0.137
Emerging Markets	0.294	0.009	-0.02	-0.071	-0.09
Equity Market Neutral	0.294	0.192	0.092	0.019	0.042
Event Driven	0.34	0.147	0.031	0.002	-0.038
Fixed Inc Arb	0.408	0.099	0.025	0.072	0
Global Macro	0.055	0.046	0.082	-0.099	0.237
Long/Short	0.159	0.06	-0.045	-0.084	-0.178
Managed Futures	0.053	-0.099	-0.009	-0.019	-0.032

We see that some hedge fund indices, in particular the Convertible Arbitrage and Fixed Income Arbitrage indices, exhibit highly significant positive autocorrelation (see figure 3).

High kurtosis and autocorrelation means that not only there are left fat tails but also that the possible big losses are correlated, so there is some chance of losing a large amount in one go.

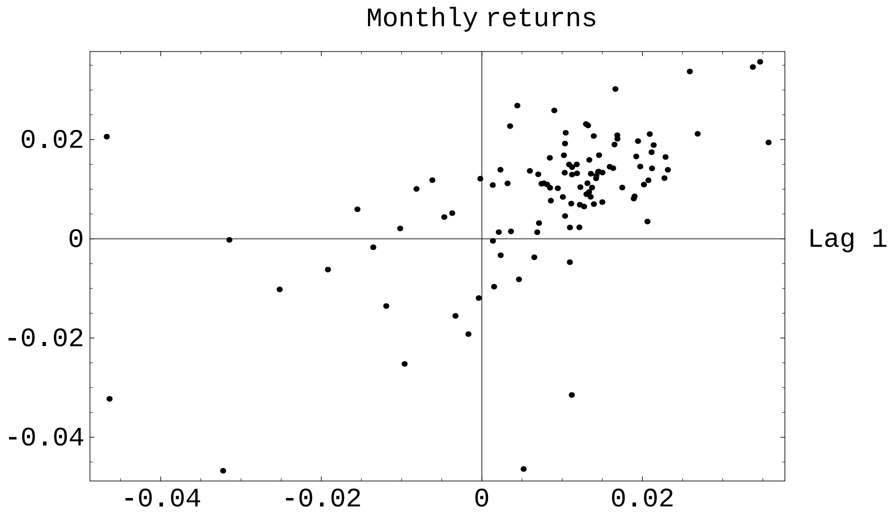

Figure 3: Autocorrelation Convertible Arbitrage

Finally, it is interesting to see how the different strategies are correlated between themselves and with the market (see Table 3). It is clear that hedge fund indices have a very low correlation with bonds and in general, correlation with equities appears to be low. Long/Short strategies have the highest correlation with S&P 500 (0.59 and 0.56 respectively), while Dedicated Short Bias is negatively correlated almost with everything and in particular with S&P 500 (-0.77). The low correlation with the market has induced investors to include hedge funds in their portfolios to increase diversification while preserving mean return. But here again we have to be careful since only under the normality hypothesis we can conclude that zero correlation means independence. In a non normal world, zero correlation is not equivalent to the independence in the movements of the assets.

Table 3: Correlation between indices

	S& P 500	JPMTUS	CSFB	C.A.	DSB	EM	EMN	ED	FIA	GM	LS	MF
S&P 500	1.00											
JPMTUS	-0.11	1.00										
CSFB	0.48	0.10	1.00									
Conv. Arb.	0.13	-0.01	0.40	1.00								
Ded. Short	-0.77	0.14	-0.48	-0.23	1.00							
Emerging Mkt	0.49	-0.17	0.65	0.36	-0.56	1.00						
Equity Mkt. N.	0.40	0.03	0.32	0.30	-0.41	0.25	1.00					
Event Driven	0.56	-0.14	0.65	0.59	-0.61	0.71	0.38	1.00				
Fixed Inc. Arb.	0.03	0.05	0.44	0.54	-0.07	0.31	0.06	0.38	1.00			
Global Macro	0.23	0.23	0.86	0.29	-0.11	0.41	0.19	0.36	0.46	1.00		
Long/Short	0.59	0.00	0.78	0.25	-0.75	0.59	0.34	0.66	0.19	0.42	1.00	
Managed Futures	-0.25	0.36	0.07	-0.30	0.29	-0.16	0.16	-0.27	-0.13	0.24	-0.10	1.00

3. Johnson's distributions.

Given a continuous random variable X whose distribution is unknown and is to be approximated, Johnson (1949) proposed a set of normalizing translations. These translations have the following general form

$$Z = \gamma + \delta \cdot g\left(\frac{X - \xi}{\lambda}\right) \tag{1}$$

where Z is a standard normal random variate, γ and δ are shape parameters, λ is a scale parameter, ξ is a location parameter and $g(-)$ is one of the following functions, each one defining a family of distributions:

$$g(y) = \begin{cases} ln(y), & \text{lognormal distribution} \\ ln\left(y + \sqrt{y^2+1}\right), & S_U \text{ unbounded distribution} \\ ln\left(y/(1-y)\right), & S_B\text{, bounded distribution} \\ y, & \text{normal distribution} \end{cases} \tag{2}$$

While the S_U distributions are defined in an unlimited range in both directions, for the bounded distributions the variate is bounded in both directions. After estimating parameters, the calculation of quantile or tail probability is simple, because these distributions come from a simple transformation of a normal distribution.

Lets consider first the S_U translation function

$$g(y) = ln\left(y + \sqrt{y^2+1}\right) = sinh^{-1}(y). \tag{3}$$

Hence,

$$Z = \gamma + \delta \cdot sinh^{-1}\left(\frac{X - \xi}{\lambda}\right) \tag{4}$$

where λ must be positive. The shape of the distribution of X depends only on the parameters γ and δ, so the distribution of the variate $Y = \frac{X-\xi}{\lambda}$ has the same shape as that of X, and we can write

$$Z = \gamma + \delta \cdot sinh^{-1}(Y). \tag{5}$$

Johnson's S_U-distribution can cover a wide range of skewness and kurtosis values. In fact, Johnson constructed tables in which he computes γ and δ in terms of skewness and kurtosis. The expected value and the lower central moments of Y are given by the following equations:

$$\mu_1'(Y) = \omega^{\frac{1}{2}} sinh\,\theta \tag{6}$$

$$\mu_2(Y) = \frac{1}{2}(\omega - 1)(\omega\, cosh(2\theta) + 1) \tag{7}$$

$$\mu_3(Y) = -\frac{1}{4}\omega^{\frac{1}{2}}(\omega - 1)^2(\omega(\omega + 2) sinh(3\theta) + 3 sinh\,\theta) \tag{8}$$

$$\mu_4(Y) = -\frac{1}{8}(\omega - 1)^2(\omega^2(\omega^4 + 2\omega^3 + 3\omega^2 - 3) cosh(4\theta) + 4\omega^2(\omega + 2) cosh(2\theta) + 3(2\omega + 1)) \tag{9}$$

where $\omega = exp(\delta^{-2})$ and $\theta = \gamma/\delta$. Observe that when $\theta = 0$ we have $\mu_3(Y) = 0$ and so the distribution is symmetric. Note also that $\omega > 1$ and μ_3 has opposite sign to γ. The skewness and kurtosis of Y, which we denote respectively as $\sqrt{\beta_1}$ and β_2, are given by:

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}}, \qquad \beta_2 = \frac{\mu_4}{\mu_2^2} \tag{10}$$

Knowing our target values for skewness and kurtosis for the variate Y, the problem is to obtain estimates the parameters γ and δ. This can be done in different ways. We can use the tables computed by Johnson, but these are limited and often need second order interpolation techniques. Another possibility is to use equations (7)-(10) to obtain estimates for γ and δ. The efficiency of this method will depend on the rate of convergence of the algorithm used to find a solution to the set of equations. Some algorithms for approximating these solutions have been given by Hill,Hill& Holder (1976). It should be noted that the behavior of these functions is quite unstable, as we can see, for example, from the plot and contour plot of skewness in terms of the values of γ and δ (Figures 4 and 5).

There is also a method of quantile estimation introduced by Wheeler (1980). This procedure gives estimators as good as the moments estimators and is usually easier to handle. The general idea is that, using the relation $w = exp((z - \gamma)/\delta)$, we associate to a conveniently chosen set of points $-z_n, -\frac{1}{2}z_n, z_0, \frac{1}{2}z_n, z_n$, a set of corresponding points in the variable X: x_p, x_k, x_0, x_m, x_n. The fact that any quantity of the form

$$\frac{x_i - x_j}{x_r - x_s} = \frac{g^{-1}(w_i) - g^{-1}(w_j)}{g^{-1}(w_r) - g^{-1}(w_s)} \tag{11}$$

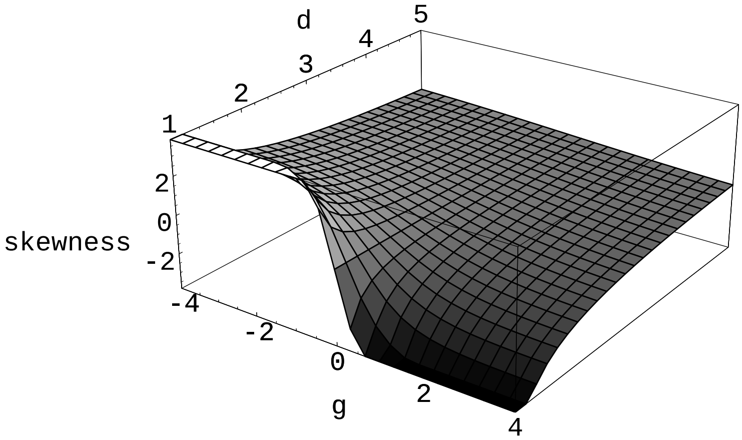

Figure 4: Skewness of S_U distribution as a function of γ and δ

does not depend on ξ or λ is used to express γ and δ in terms of the five quantities x_p, x_k, x_0, x_m, x_n and then sample order statistics are substituted to obtain estimates $\hat{\gamma}$, $\hat{\delta}$. So first we have to obtain the sample statistics: $\hat{x}_p, \hat{x}_k, \hat{x}_0, \hat{x}_m, \hat{x}_n$. These five points serve to approximate the shape of the S_U distribution, which is summarized by the following statistics:

$$t = \frac{x_n - x_0}{x_0 - x_p} \qquad t_u = \frac{x_n - x_p}{x_m - x_k} \tag{12}$$

Observe that the statistic t depends on the symmetry of the curve since it compares the two tail lengths. On the other hand, t_u gives the relative length of the tails compared to the central part of the distribution.

Defining $a = exp(-\gamma/\delta)$ and $b = exp(\frac{1}{2}z_n/\delta)$ we obtain the equations

$$a^2 = \frac{1 - tb^2}{t - b^2}, \qquad b = \frac{1}{2}t_u + \sqrt{(\frac{1}{2}t_u)^2 - 1} \tag{13}$$

The only conditions imposed for the existence of solutions are that $b^2 > t > b^{-2}$ and $t_u > 2$. We then obtain the estimators:

$$\hat{\delta} = \frac{1}{2}z_n/ln(b), \qquad \hat{\gamma} = -\hat{\delta}ln(a) \tag{14}$$

Once we have these estimates for Y, we can make adjustments of scale and location. Since $Y = \frac{x-\xi}{\lambda}$, where X is the variate with whose distribution we are concerned, then if the

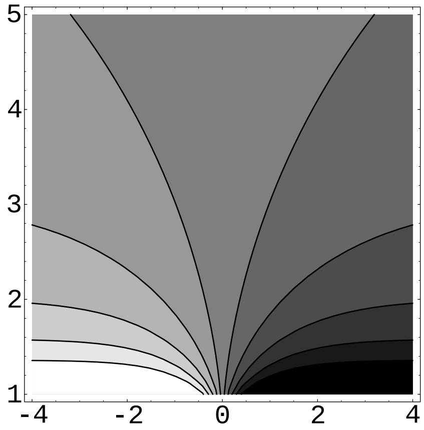

Figure 5: Skewness contour plot

two first moments of Y are to be identical with those of X, we have:

$$\lambda = \sigma(x)/\sigma(y), \qquad \xi = E(X) - \lambda E(Y). \tag{15}$$

The distribution function of a S_U distributed variate X is given by the equation:

$$f_X(x) = \frac{\delta}{\lambda\sqrt{2\pi\left((\frac{x-\xi}{\lambda})^2+1\right)}} exp\left[-\frac{1}{2}\left(\gamma + \delta \cdot sinh^{-1}\left[\frac{x-\xi}{\lambda}\right]\right)^2\right] \tag{16}$$

Before we turn to the next section it is important to recall that S_U distributions, although they cover a wide range of values for skewness an kurtosis, they do not cover the whole $(\sqrt{\beta_1}, \beta_2)$ plane. In fact, they only cover the region which is above the points of the lognormal line (the line whose points correspond to lognormal distributions). The lognormal line is given by the parametric equations

$$\beta_1 = (\omega - 1)(\omega + 2)^2 \quad (\sqrt{\beta_1} > 0) \tag{17}$$

$$\beta_2 = \omega^4 + 2\omega^3 + 3\omega^2 - 3. \tag{18}$$

For any thicker tail distribution than a lognormal, there is an appropriate S_U distribution, but pairs of values $(\sqrt{\beta_1}, \beta_2)$ below the lognormal line, we have to consider the S_B system of distributions. Unfortunately, for this type of distributions there are no simple expressions of the higher moments in terms of γ and δ, so we have to rely on Johnson's tables or apply numerical approximations.

4. Fitting HF indices with Johnson distributions

We will know apply the above to the case of CSFB/Tremont indices. Looking back at Table 1, the first question is to decide which distribution shall we use for the given values of skewness and kurtosis. We find that those indices with more non-normal behavior, Event Driven and Fixed Income Arbitrage, are precisely the indices which we cannot approximate with S_U distributions, since they appear considerably below the log-normal line defined by equations (17) and (18). On the other hand, Equity Market Neutral is almost on the lognormal line, as we can see in figure (6). In fact most of the indices appear close to the lognormal line, something which suggests that the choice of the appropriate distribution can be quite unstable.

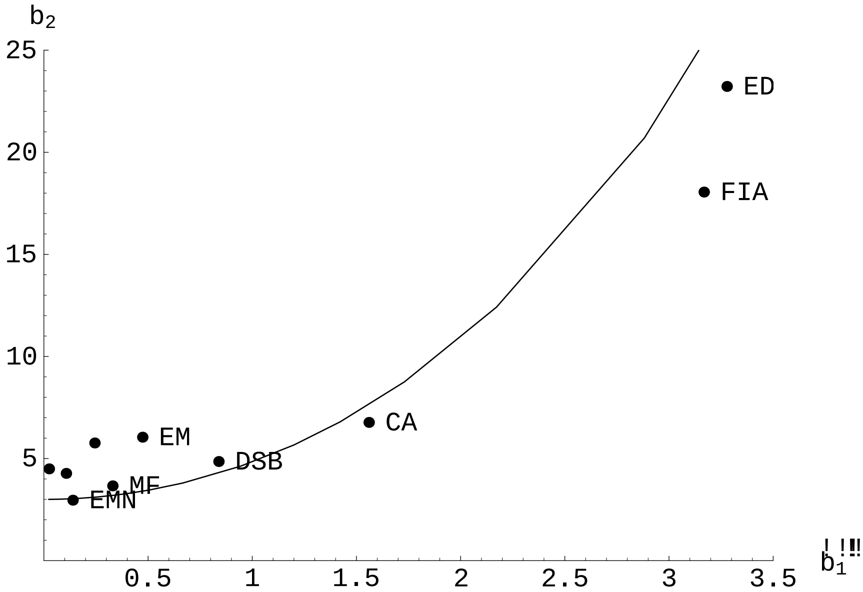

Figure 6: HF indices and log-normal line

According to the above, we will approximate the Event Driven and Fixed Income Arbitrage indices using S_B-distributions. In these cases we estimated the parameters γ and δ performing numerical approximations with *Mathematica*. Table 4 displays the values obtained for the parameters γ and δ for each of the CSFB/Tremont hedge fund indices, indicating the Johnson translation system used in each case:

Table 4: γ and δ of HF indices

	$E(x)$	st. dev	Skewness	Kurtosis	Distribution	γ	δ
CSFB/Tremont Index	0.88%	2.56%	0.107	4.27	S_U	-0.12	2.12
Convertible Arbitrage	0.84%	1.41%	-1.56	6.77	S_U	1.1153	1.6282
Ded Short Bias	0.18%	5.29%	0.839	4.85	S_U	-1.56	2.41
Emerging Markets	0.54%	5.34%	-0.473	6.04	S_U	0.302	1.656
Equity Mkt. Neutral	0.87%	0.92%	0.139	2.9599	*lognormal*	-	-
Event Driven	0.86%	1.81%	-3.279	23.225	S_B	-4.565	1.108
Fixed Inc Arb	0.55%	1.18%	-3.168	18.05	S_B	-4.045	1.089
Global Macro	1.18%	3.65%	-0.025	4.495	S_U	0.0255	2.006
Long/Short	0.96%	3.31%	0.245	5.7631	S_U	-0.159	1.665
Managed Futures	0.62%	3.49%	0.03348	3.663	S_U	-0.0751	2.736

As an example, in figures 7 and 8 we plot the density curves of S_B distributions which approximate the empirical distribution of Event Driven and Fixed Income Arbitrage indices.

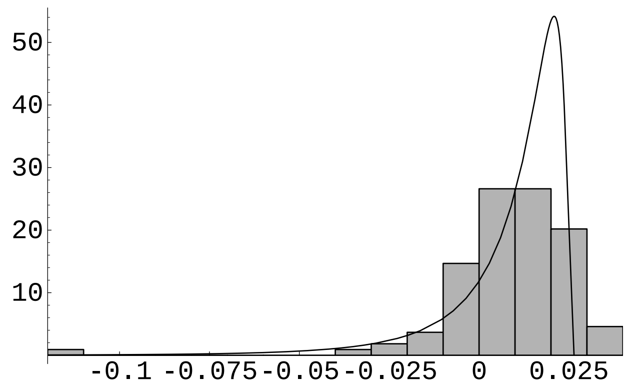

Figure 7: Event Driven S_B-distribution

5. Testing the Johnson approximation with the Omega parameter

In order to define a measure of profit/loss which takes into account the higher moments of a distribution of returns, Con Keating and William Shadwick (2001) defined a statistic

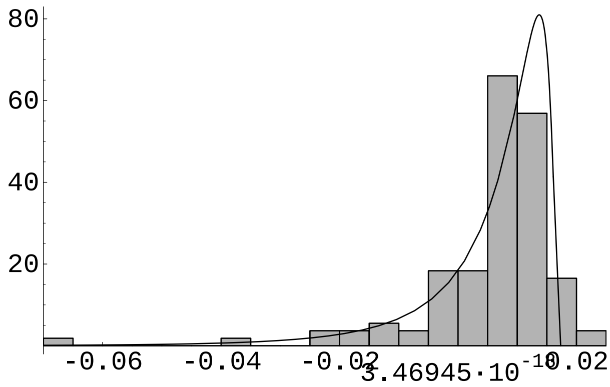

Figure 8: Fixed Income Arbitrage S_B-distribution

which can help investors to visualize the risk and return in a simple way. This statistic is called *Omega* and can be calculated from the cumulative distribution function $F(x)$ as the quotient

$$\Omega(L) = \frac{\int_L^b 1 - F(x)dx}{\int_a^L F(x)dx} \tag{19}$$

where L is a loss threshold defined by the investor, and (a, b) is an interval containing the range of returns. Hence $\Omega : (a, b) \to (0, \infty)$ is a monotone decreasing function of the return level L. This function allows us to compare returns for different assets and to rank them according to the value of $\Omega(L)$ for different L. But it can also be used as a tool to compare a theoretical distribution with the empirical distribution. Since we can compute $\Omega(L)$ directly from the discrete return observations and also from the corresponding Johnson approximation we will use it as a measure of how well is the Johnson distribution reflecting the behavior of returns.

Hence, we first consider the empirical distribution associated to each HF index and we compute the value of Ω for different values of the parameter L around zero. In Table 5 we show these results obtained together with the corresponding Sharpe ratios as provided by CSFB Tremont.

Table 5: Omega and Sharpe ratios of empirical distribution

	Sharpe*		Omega(L)				
			L=-0.002	L=-0.001	L=0	L=0.001	L=0.002
CSFB Tremont	0.071		1.977	1.796	1.610	1.610	1.610
Convertible Arbitrage	1.24		2.794	2.393	2.052	1.694	1.455
Ded Short Bias	-0.22		1.015	0.877	0.877	0.811	0.761
Emerging Markets	0.02		1.195	1.195	1.001	0.935	0.935
Equity Market Neutral	2.07		6.600	4.948	3.796	3.363	2.140
Event Driven	0.99		2.756	2.310	1.966	1.663	1.403
Fixed Inc Arb	0.57		2.379	2.013	1.515	1.152	0.872
Global Macro	0.78		1.981	1.764	1.657	1.657	1.657
Long/Short	0.62		2.256	1.738	1.738	1.435	1.364
Managed Futures	0.21		1.309	1.221	1.111	1.026	0.911

*Calculated using the rolling 90-day T-bill rate.

We have evaluated Omega near zero in order to be able to compare with the risk adjusted returns. Figures 7 and 8 show the behavior of the different strategies. It should be noted the greater steepness of the Event Driven and Fixed Income Arbitrage indices, which reflects the fact that for these indices, a small increase in the investor's loss threshold quickly reduces their Omega.

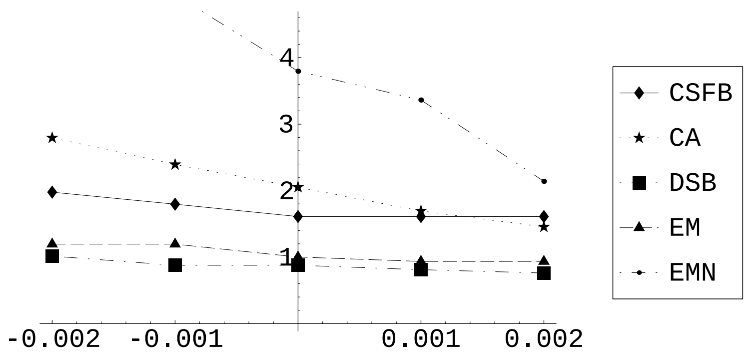

Figure 9: $\Omega(L)$ for empirical series

If we rank the hedge fund indices according to their Sharpe ratio or to Ω parameter we find out significant differences precisely in those indices with low skewness and high

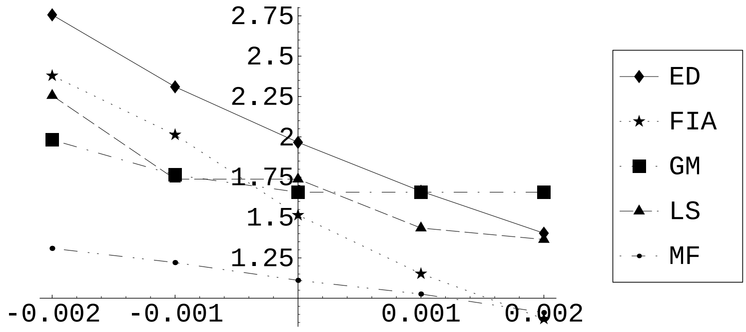

Figure 10: $\Omega(L)$ for empirical series

kurtosis. In Table 6 we compare the rankings obtained from a risk adjusted measure, from the Sharpe ratio and from the Omega parameter (at a level of L=0.002).

Table 6: Ranking by Sharpe ratios and by Omega ($L = 0.002$)

Risk adjusted	Ranking Sharpe*		Ranking Omega**
Equity Market Neutral	Equity Market Neutral		Equity Market Neutral
Convertible Arbitrage	Convertible Arbitrage		Global Macro
Event Driven	Event Driven		CSFB Tremont
Fixed Inc Arb	Global Macro		Convertible Arbitrage
CSFB Tremont	CSFB Tremont		Event Driven
Global Macro	Long/Short		Long/Short
Long/Short	Fixed Inc Arb		Emerging Markets
Managed Futures	Managed Futures		Managed Futures
Emerging Markets	Emerging Markets		Fixed Inc Arb
Ded Short Bias	Ded Short Bias		Ded Short Bias

*Calculated with the rolling 3-month T-bill. ** for $L = 0.002$.

The results agree with what we expected: the very low skewness and high kurtosis of the Event Driven and Fixed Income Arbitrage indices are penalized under the Omega criterion at any return level which is slightly positive. At $L = 0.002$ they both descend more than one position. It is also clear that as L increases, both indices will descend even more. This confirms that their Sharpe ratio, which ignores higher moments, was overestimating the performance of these indices. It is also significant that Equity Market Neutral index, for which we can assume the normality hypothesis, appears in a stable first position both under the Sharpe ratio and the Omega criterion.

We will now calculate Omega for each of the Johnson's distributions we have associated to each of the indices. The cumulative distribution function of the corresponding S_U and S_B distributions and the integrals needed in (19) were obtained by numerical approximation in *Mathematica*. We obtain the following values:

Table 7: Omega for the Johnson distributions

	Omega(L)		
	L=-0.002	L=0	L=0.002
CSFB Tremont	3.03	2.4687	2.011
Convertible Arbitrage	5.7828	4.2835	3.1259
Ded Short Bias	1.2048	1.09	0.988
Emerging Markets	1.45042	1.3131	1.19528
Equity Market Neutral	21.9404	11.8382	6.538
Event Driven	4.18	3.34	2.62
Fixed Inc Arb	4.49	3.19	2.19
Global Macro	2.683	2.32734	2.017
Long/Short	2.77	2.1948	2.065
Managed Futures	1.8318	1.5806	1.363

We can now compare the ranking obtained with these values. In Table 8 we also rank the indices according to the steepness of their omega around zero. Recall that the more steepness, the less risky.

Table 8: Ω-Ranking using Johnson distributions

Ranking omega		$\Delta\Omega/\Delta L$
$L = 0$	$L = 0.002$	
Equity Market Neutral	Equity Market Neutral	Equity Market Neutral
Convertible Arbitrage	Convertible Arbitrage	Convertible Arbitrage
Event Driven	Event Driven	Fixed Income Arb.
Fixed Income Arb	Fixed Income Arb.	Event Driven
CSFB Tremont	Long/Short	CSFB/Tremont
Global Macro	Global Macro	Global Macro
Long Short	CSFB/Tremont	Managed Futures
Managed Futures	Managed Futures	Long/Short
Emerging Markets	Emerging Markets	Emerging Markets
Ded Short Bias	Ded Short Bias	Ded Short Bias

We can see that the ranking is similar to the one given by the simple risk/return ratio. In fact, at $L = 0$ both rankings coincide while at $L = 0.002$ there are still 8 coincidences which include in particular the Event Driven and Fixed Income Arbitrage indices. This can be seen more clearly if we compare the Omega function obtained from the discrete observations with the one obtained from the Johnson distribution. Around $L = 0$, the Johnson distributions appear to have higher Omega values than the corresponding empirical distributions.

6. Conclusions.

We have seen that although Johnson distributions are a useful tool to model hedge fund non-normal distributions of returns, there are still some considerations which should be taken into account. First, on the practical side, the parameter estimation depends heavily on numerical approximations and thus is more prone to error. Moreover, as we approach the lognormal line in the $(\sqrt{\beta_1}, \beta_2)$ plane there may be some instability on the choice of distribution and the estimation of the γ and δ parameters. On the other hand, under the Omega criterion it appears that, although the Johnson approximation carries information about skewness and kurtosis, it still fails to reflect entirely higher moment effects. It also fails to detect entirely the the left tail risk, specially in the cases where we have a "short option" behavior. In these cases, the left tail of the associated Johnson distribution underestimates the probability of losses. The isolated but large negative returns seem to be too rare to make any difference on the Johnson parameters, and hence they remain undetected.

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