Intra- and Inter-sectoral Knowledge Spillovers and TFP Growth Rates

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Abstract. - In this paper I estimate unobserved labor-generated knowledge spillovers within and between six large macroeconomic sectors covering the US civilian economy from 1948 to 1991 and relate them to observed productivity changes. I construct a series of sectoral knowledge spillover matrices that show the changes in the magnitude and direction of intra- and inter-sectoral spillovers for each sector. I show that the productivity slowdown in the US economy of the early seventies is associated with a decline of intra-sectoral spillovers and the emergence of inter-sectoral spillovers. This change coincides with trade taking over manufacturing as the main source and destination of new knowledge flows. The analysis of technology flows, measured as the production and use of patents, corroborates this finding. Furthermore, I also compute the gap between the market, which ignores knowledge spillovers, and the optimal allocation of labor across sectors, and the wedge between market and optimal wage rates by sector. I show that optimal employment in manufactures is 32% higher than the market allocation, and that optimal wages in the overall economy are 31% above market wages.

Keywords: Knowledge spillovers; technology; productivity slowdown.
JEL Classification: D24, J24, O30, O40.
1 Introduction

Although spillovers have always had an important role in economic theory and policy design, the empirical estimation of their magnitude and the extent of their contribution to productivity changes has not been as popular. Moreover, the amount of literature dedicated to knowledge spillovers is far inferior to the number of studies that focus on the measurement of R&D investment-generated spillovers, in particular of localized spillovers derived from Marshallian agglomeration economies, or on the difficulty in appropriating the benefits of one’s own innovative activity. Also, because of data availability and/or quality, most measurements of external economies refer to U.S. and European manufacturing.

In this paper I estimate labor-generated knowledge spillovers within and between six large macroeconomic sectors covering the whole U.S. civilian economy. I gauge whether these spillovers are related to observed productivity changes, and how they affect sectoral total factor productivity (TFP) growth rates. I then compute the gap between the market and the optimal sectoral allocation of labor, the spillover-generating input, and its return rates. The market allocation of and rates of return to all other factors of production are already efficient.

I find more labor should be allocated to the main spillover generating sector, manufacturing, so that employment in this sector ought to increase by 32%, and output by 8%, while optimal wages in the overall economy ought to be 31% above what the market signals. In particular, wages in all sectors, except for mining, ought to be at least 10% above market return rates. Clearly, my estimates are at their most robust when considered ordinally, in their ranking and relative weights rather than in their absolute values.

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2 Usually, firms that invest more in R&D are supposed to benefit more from knowledge externalities. But according to Cockburn and Henderson (1998) causality works in the opposite direction: absorbing knowledge spillovers allows a firm to invest more in R&D.

3 See Döring and Schnellenbach (2006) for a recent survey of the literature on localized knowledge spillovers.
I also find that the productivity slowdown of the early seventies coincides with trade taking over manufacturing as the main generator of knowledge spillovers to the whole economy and with increasing sectoral integration, that is, with spillovers within sectors declining in favor of spillovers between sectors. Interestingly, the productivity slowdown does not seem to affect much the efficiency gap, so that the wedge between the market and the optimal sectoral allocation of labor and the corresponding return rates do not experience a noticeable change after 1973.

My approach introduces a twofold novelty: first, spillovers are generated by the quality of the overall human capital employed, not just by those workers involved in R&D activities, and, second, spillover estimates cover all sectors in the economy, not only the manufacturing industries. Here, knowledge spillovers in a particular industry are generated by the capacity of its employees at all stages and levels of the production process to learn from their own and from others’ productive experience. In other words, because employees learn by observing as well as by doing, workers absorb (i.e. recognize, adopt and adapt) flows of knowledge regardless of their origin, be they blueprints, managerial techniques, new organizational designs or be they embodied in new capital equipment or intermediate goods.

The concept of labor-led knowledge spillovers feeds partly on the labor literature but, mostly, on studies of how the mechanics of the learning process affect productivity. Arrow (1962) was among the first to consider the economic impact of learning from experience and to formally model spillovers of any kind. More recently, Jovanovic and Rousseau (2002) have extended his arguments to the “new economy.” Among others, Lieberman (1984), Cohen and Levinthal (1990), Jovanovic and Nyarko (1995), and Klenow (1998) have also worked along the same lines.

In this paper I start from an index number approach in a production theoretic framework and go on to propose a static model, which is extendable to a dynamic setting. The estimation of spillovers proceeds in two stages: first, I compute sectoral TFP growth as the difference between the rates of growth of output and inputs using
a Tornqvist Index; in the second stage I recover the spillover parameters, identified by \textit{observed} productivity changes, by a constrained least squares procedure. I use the database developed by Dale W. Jorgenson for the US civilian economy from 1948 through 1991 containing quality-adjusted factor and product sectoral panel data. The use of this database allows me to distinguish between substitution among different types of inputs (with different combinations of marginal productivity in their components) and growth in productivity. Often, what previous studies have called spillovers were really input quality improvements.\footnote{In reference to input quality improvement and the contribution of inputs to economic growth see Jorgenson, Gollop and Fraumeni (1987), Jorgenson, Ho and Fraumeni (1994), and Jorgenson and Stiroh (1994).} Once input heterogeneity and quality changes are taken into account, the TFP term will only pick up the \textit{costless} spillover effects.

I provide, first, a matrix of origin and destination of knowledge spillovers within and between six large macroeconomic sectors: Manufacturing, Mining, Construction, Services, Trade & Transportation, and Agriculture. Arguably, both the sectoral composition and the technological distribution of firms and industries are characteristics that define an economy. Both vary through time, and both determine the labor or human capital distribution within and across sectors. Therefore, the matrix of intra- and inter-sectoral knowledge spillovers is specific to each economy and period.

Second, I examine the nature of knowledge flows, that is, whether knowledge is mostly embodied in the spillover generating input, labor, and, hence, spills over through workers transiting between sectors, or whether it is mostly disembodied and, therefore, technology flows can be a good proxy for knowledge spillovers in terms of direction (origin and destination) and relative size. To this purpose I compare, first, my matrix of spillover estimations with a matrix of the economy’s transitional labor flows, and then, with a matrix of patent expenditure by sector of origin and sector of use. I find labor-generated knowledge flows follow more closely the patterns of expenditure and use of patents and R&D than the transitional labor flows.
Third, I examine how variations in the relative contribution of sector-specific and inter-sectoral spillovers to the total spillover change in reflection of the productivity slowdown of 1973. During the whole 1948-1991 period Manufacturing was the leading knowledge generator for the whole economy, but the productivity slowdown coincides with a decline in intra-sectoral spillovers and the rise of Trade & Transportation as the main source of knowledge spillovers. And, fourth, I find efficiency requires allocating more resources into the main spillover generating sector. Thus, for the whole period, it would be optimal for the market to increase the number of those employed in Manufacturing by 32%, so that output would increase by 8% and wages in all sectors except Mining by at least 10%.

The remainder of the paper is organized as follows. The next section describes a model with inter- and intra-industry labor-generated knowledge spillovers and states the difference between the market and the optimal solutions in terms of resource allocation and return to labor; Section 3 goes through the estimation procedure to measure these spillovers. Section 4 describes the dataset used, as well as the criteria chosen to aggregate industries into six large macroeconomic sectors, and the literature-based discretionary choices for the magnitude and ranking of sectoral spillovers. Section 5 discusses the resulting matrices of knowledge flows and compares them to the matrices of labor and technology flows. Section 6 interprets the productivity slowdown in terms of changing patterns of knowledge flows. Section 7 reexamines the gap between the market and the optimal solution in light of the data. Finally, Section 8 summarizes the paper’s main conclusions.

2 Model

This section presents a model where the production function incorporates knowledge externalities in labor. I show that the presence of spillovers leads to a difference between the competitive and the optimal solution in terms of labor’s sectoral allocation
Environment

Consider an economy consisting of \( n \) sectors, each producing a differentiated final good \( Y_i \) with capital \( K_i \), labor \( L_i \), and intermediate goods \( M_i \). The production function at any given period is characterized by sectoral knowledge spillovers in labor, that is, 

\[
Y_i = A_i K_i^{\beta_i K} L_i^{\beta_i L} M_i^{\beta_i M} \prod_{j=1}^{n} L_j^{\gamma_{ij}}, \quad i = 1, 2, \ldots, n.
\]

The exogenous time-invariant scale factor \( A_i \) is here unrelated to the input variables and, hence, there is no endogenous growth derived from it.\(^5\) The \( L_j^{\gamma_{ij}} \) are sectoral spillovers that improve the marginal productivity of all inputs in the sector equally and costlessly. They are characterized by the learning parameters \( \gamma_{ij} \geq 0 \) that measure the extent to which sector \( i \) learns from sector \( j \). If \( i = j \), they are called sector-specific or intra-sectoral knowledge spillovers; if \( i \neq j \), they are inter-sectoral knowledge spillovers. Every sector \( i \) exhibits constant returns to scale \( \beta_i K + \beta_i L + \beta_i M = 1 \). Endowments are fully used: \( K = \sum_{i=1}^{n} K_i, \quad L = \sum_{i=1}^{n} L_i, \) and \( M = \sum_{i=1}^{n} M_i \). The consumer’s utility function is \( U(C_1, C_2, \ldots, C_n) = \prod_{i=1}^{n} C_i^{\alpha_i} \), where \( \sum_{i=1}^{n} \alpha_i = 1 \) and \( \alpha_i > 0 \quad \forall i \).

Each sector \( i \) receives total spillover \( q_i = \sum_{j=1}^{n} \gamma_{ij} \) and emits total spillover \( \Gamma_i = \sum_{j=1}^{n} \alpha_j \gamma_{ji} \). The economy-wide coefficient for each factor \( X = K, L, M \) is \( \beta_X = \sum_{j=1}^{n} \alpha_j \beta_{jX} \) and the economy-wide emission (and reception) of spillovers is \( Q = \sum_{j=1}^{n} \alpha_j q_j = \sum_{i=1}^{n} \Gamma_i \).

Note that, whereas each sector \( i \) operates under the assumption of constant returns to the inputs it controls, social returns to its production function are \( 1 + q_i \) which, unless \( q_i = 0 \), means there are really sectoral and, hence, economy-wide increasing

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\(^5\)A dynamic extension of this model would have \( A_i \) as a Hicksian neutral shift parameter: the scale factor \( A_i \) would vary over time as the productivity of inputs and/or the knowledge spillovers change. In this model \( A_i \) is the ratio of output to total factor input plus spillovers.
returns to scale.\footnote{Note also that sectoral private returns to spillover-generating \( L_i \) are really \( \beta_{iL} + \gamma_{ii} \) and social returns are \( \beta_{iL} + \Gamma_i \).}

With this technology and these preferences I evaluate two possible arrangements next: a competitive market solution and the social planner’s solution. If there is a wedge between the social and the private rates of return to spillover-generating labor, TFP estimates should reflect the externality. Otherwise, the effect of the knowledge spillover will be fully accounted for and disappear from the residual.

**Market Solution**

On the production side, representative firms ignore knowledge spillovers and thus maximize profits \( P_i Y_i - w_K K_i - w_L L_i - w_M M_i \), choosing \( K_i, L_i, M_i \) by setting each input’s marginal product equal to its return rate:

\[
w^*_X = \beta_{iX} \frac{P_i Y_i}{X_i}, \quad X = K, L, M.
\]

(1)

Assuming that all inputs are perfectly mobile across sectors and indifferent among them there is a unique competitive return rate for each input. Thus, using sector 1 as the numeraire, \( P_1 = 1 \):

\[
P_i = \frac{\beta_{1X} X_i Y_i}{\beta_{iX} X_1 Y_1}.
\]

(2)

A competitive equilibrium is attained at zero profits for each sector: \( P_i Y_i - w_K K_i - w_L L_i - w_M M_i = 0 \), which implies that the value of total production equals the sum of the input values\footnote{This “product exhaustion” also implies that the value shares of all inputs sum up to one.} or consumers’ income: \( \sum_{i=1}^{n} P_i Y_i = w_K K + w_L L + w_M M = Y \).

On the demand side, the representative consumer chooses consumption goods \( C_1, C_2, ...C_n \) to maximize her utility \( U(C_1, C_2, ...C_n) \) subject to \( Y = \sum_{i=1}^{n} P_i C_i \). Consumers also ignore knowledge spillovers in deciding on their consumption.
order condition, with \( P_1 = 1 \), yields

\[
P_i = \frac{\alpha_i}{\alpha_1} \frac{C_1}{C_i}.
\]

Setting supply equal to demand, \( C_i = Y_i \) and combining this equation with Equation (2) yields \( X_i = \frac{\alpha_i}{\alpha_1} \frac{\beta_i X}{\beta_1 X} X_1 \), which together with \( X = \sum_{i=1}^n X_i \) implies that the market’s sectoral allocation of inputs is

\[
\frac{X_i^c}{X} = \frac{\alpha_i \beta_i X}{\beta X}, \quad X = K, L, M.
\]

Hence, the market completely ignores the existence of labor spillovers: the competitive allocation of inputs, including labor, is determined exclusively by consumers’ preferences and technology parameters. A sector’s derived demand for an input, i.e. the sectoral input share in the economy, is a product of two shares: the sector’s share in the economy \( \alpha_i \) and the relative input share in that sector \( \frac{\beta_i X}{\beta X} \).

**Optimal Solution**

The social planner, on the other hand, internalizes knowledge spillovers and chooses the \( L_i \) for each sector \( i \) that maximize the representative consumer’s utility \( U(Y_1, ..., Y_n) \) subject to \( K = \sum_{i=1}^n K_i, L = \sum_{i=1}^n L_i, \) and \( M = \sum_{i=1}^n M_i \). The first order conditions for capital and intermediate goods are

\[
\frac{\alpha_1 \beta_1 X}{X_1} = \frac{\alpha_i \beta_i X}{X_i},
\]

which imply

\[
\frac{X_i^s}{X} = \frac{\alpha_i \beta_i X}{\beta X}, \quad X = K, M.
\]
Thus, because of Cobb-Douglas preferences and technology, the planner allocates capital and intermediate goods exactly as the market does. However, the first order condition for labor is

\[
\frac{\alpha_1 \beta_1 L}{L_1} + \sum_{j=1}^{n} \frac{\alpha_j \gamma_{jj}}{L_1} = \frac{\alpha_i \beta_i L}{L_i} + \sum_{j=1}^{n} \frac{\alpha_j \gamma_{ji}}{L_i},
\]

which implies

\[
\frac{L^*_i}{L} = \frac{\alpha_i \beta_i L + \Gamma_i}{\beta L + Q}.
\]

Thus, the optimal allocation of labor does take into account knowledge spillovers. The optimal allocation of labor to a sector \( i \) depends, as with the market, on consumers’ preferences for what this sector produces, that is, on the sector’s share of the economy \( \alpha_i \), and on the sector’s relative elasticity of labor with respect to the overall economy’s. But now, it also depends on sector \( i \)’s relative emission of knowledge spillovers to the whole economy.

**Proposition 1** The planner allocates more labor than the market to sector \( i \), \( L^*_i > L^*_i \), iff \( \frac{\Gamma_i}{Q} > \frac{\alpha_i \beta_i L}{\beta L} \). Similarly, \( L^*_i < L^*_i \), iff \( \frac{\Gamma_i}{Q} < \frac{\alpha_i \beta_i L}{\beta L} \) and \( L^*_i = L^*_i \), iff \( \frac{\Gamma_i}{Q} = \frac{\alpha_i \beta_i L}{\beta L} \). Proof:

\[
L^*_i \leq L^*_i \text{ if } \frac{\alpha_i \beta_i L + \Gamma_i}{\beta L + Q} \leq \frac{\alpha_i \beta_i L}{\beta L}, \text{ which is equivalent to } \frac{\Gamma_i}{Q} \leq \frac{\alpha_i \beta_i L}{\beta L}.
\]

That is, the planner’s allocation of labor to sector \( i \) is larger (smaller, equal) than the market’s only if sector \( i \)’s relative emission of spillovers \( \frac{\Gamma_i}{Q} \) is larger (smaller, equal) than its sectoral labor share in the economy \( \frac{\alpha_i \beta_i L}{\beta L} \).

When there are no inter-sectoral spillovers, that is, when \( \gamma_{ij} = 0 \), the planner’s allocation becomes

\[
\frac{L^*_i}{L} = \frac{\alpha_i (\beta_i L + \gamma_{ii})}{\sum_{j=1}^{n} \alpha_j (\beta_{jL} + \gamma_{jj})}.
\]
Clearly, the planner allocates more labor to the sector with the largest intra-sectoral spillover. If relative learning parameters are equal across sectors, i.e. \( \frac{\gamma_{ii}}{\beta_{iL}} = \frac{\gamma_{jj}}{\beta_{jL}} \), there is no difference between the market and the planner’s allocation of labor.\(^8\)

Figure 1 illustrates the differences between the market and the planner’s allocation of labor in a simple two-sector economy in which \( \beta_{1L} = \beta_{2L} = \beta_L \) and \( q_1 > q_2 \). The solid black 45° line represents the market allocation to sector 1 according *exclusively* to consumers’ preferences: \( \frac{L_1^c}{L} = \alpha_1 \). The three color lines represent the planner’s allocation of labor, \( \frac{L_1^s}{L} \), according to three different sets of inter-sectoral learning parameters: the blue hollow-squared line for the case when both sectors in the economy learn *equally* from one another (\( \gamma_{12} = \gamma_{21} > 0 \)); the magenta hollow-squared line for the case when *only* the sector with the highest spillover, sector 1, learns from the other sector (\( \gamma_{12} > 0 \), \( \gamma_{21} = 0 \)); and finally, when no sector learns from the other sector’s productive experience (\( \gamma_{12} = \gamma_{21} = 0 \)) the planner allocates labor according to the green cross line. In all these cases the planner *also* takes into account how much productive knowledge sector 1 generates, whether for itself (\( \gamma_{11} \)) or for the rest of the economy (\( \gamma_{21} \)), and how much it receives from the other sector.

When \( \alpha_1 = 0 \) the market allocates no workers to sector 1, whereas the planner, as long as sector 1 generates knowledge that spills over to the the rest of the economy (\( \gamma_{21} > 0 \), as in the blue line), will always assign some workers to sector 1, the more so the more the rest of the economy learns from sector 1: \( \frac{L_1^s}{L} = \frac{\gamma_{21}}{\beta_L + q_2} > 0 \). At the other extreme, when \( \alpha_1 = 1 \), the market allocates all workers to sector 1, whereas the planner will allocate some workers to sector 2 for as long as sector 1 learns from it (i.e. the larger \( \gamma_{12} \), as in the blue and magenta lines): \( \frac{L_1^s}{L} = \frac{\beta_L + \gamma_{11}}{\beta_L + q_1} < 1 \).

In principle, both the market and the planner respond to an increase in consumers’ preferences for one good increasing the labor allocated to its production.\(^9\) However, 

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\(^8\)The social planner will consider \( \beta_{iL} + \gamma_{ii} \) as the labor coefficient for each sector \( i \). A change in the intra-sectoral learning parameter \( \gamma_{ii} \) will cause the TFP of sector \( i \) to increase, even if everything else remains unchanged, including the sectoral labor allocation and technology parameters.

\(^9\)Because the supply of labor is fixed, this also means the proportion of labor allocated to the rest of the sectors in the economy will diminish.
the planner also takes into consideration how much productive knowledge the workers employed in each sector generate for the benefit of the overall economy. If employing more workers in a particular sector will generate more knowledge for the whole economy and, thus, make the generating sector and/or other sectors more productive, the planner will allocate more workers to this sector than strictly determined by market preferences and technological needs. The market’s sectoral allocation of labor, on the other hand, will forego this productivity gains. Clearly, as long as knowledge spillovers exist, regardless of their pattern, there will be a gap between the market solution and what is optimal.

Social and Private Rates of Return to Labor

As seen above, under Cobb-Douglas preferences and technology, return rates for capital and intermediate inputs for any sector \( i \) are the same for the market and the planner. From Equations (1) and (2) we know there is a unique competitive wage rate for all sectors: \( w^c = \beta_1 L_1 \frac{Y_i}{L_i} \). The planner’s shadow wage rate for sector \( i \) measures the productivity of labor employed in that sector expressed in physical units of output of sector 1, that is, in real terms:

\[
w_i^s = Y_i L_i \frac{U_i}{U_1} = (\beta_{1L} + \gamma_{ii}) \frac{\alpha_i Y_1^s}{\alpha_1 L_1^s}
\]

where \( U_i \) is the marginal utility derived from the consumption of one extra unit of good \( i \), with \( \frac{U_i}{U_1} = \frac{\alpha_i Y_1^s}{\alpha_1 Y_i^s} \). Thus, the optimal wage rate is sector-specific: \( w_i^s \neq w_j^s \), \( \forall i \neq j \). In relation to the competitive wage rate:

\[
\frac{w_i^s}{w^c} = \left( 1 + \frac{\gamma_{ii}}{\beta_{1L}} \right) \lambda_i,
\]

where \( \lambda_i = \frac{Y_i^s}{L_i} / \frac{Y_1^s}{L_1} \) is the optimal-to-market average product of labor ratio measured in units of the numeraire. Clearly, sectoral real productivity of labor depends on each sector’s relative intra-sectoral spillover, \( \frac{\gamma_{ii}}{\beta_{1L}} \), and on the distance between its
relative spillover emission, $\frac{\Gamma_i}{Q}$, and its relative market labor allocation, $\frac{\alpha_i \beta_{iL}}{\beta_L}$, as per Proposition 1.

If we examine the last expression in more detail and rewrite it as

$$\frac{w^s_i}{w^c} = \left( 1 + \frac{\gamma_{ii}}{\beta_{i1L}} \right) \left( \frac{L^s_i}{L^c_1} \right)^{\beta_{1L}} \prod_{j=1}^{n} \left( \frac{L^s_j}{L^c_j} \right)^{\gamma_{ij}} / \left( \frac{L^c_j}{L^c_i} \right),$$

we can see, firstly, that as long as sector $i$ exhibits some degree of learning-by-doing ($\gamma_{ii} > 0$), productivity in real terms for sector $i$ will be larger with the planner’s allocation of labor.

Secondly, that there is a constant level effect for all sectors, $\left( \frac{L^s_i}{L^c_1} \right)^{\beta_{1L}}$, increasing in the numeraire’s labor elasticity of output, and in the difference between the numeraire’s relative emission of spillovers, $\frac{\Gamma_i}{Q}$, and its relative market labor allocation, $\frac{\alpha_i \beta_{iL}}{\beta_L}$, as per Proposition 1. The larger (smaller) this level effect, *ceteris paribus*, the higher (lower) real productivity for all sectors is with the planner’s allocation of labor.

And, thirdly, we can also see that the more workers the planner allocates to sector $i$ relative to the market, $L^s_i/L^c_i$, the lower the relative wage rate, $w^s_i/w^c$, will be for these workers, independently of how much productive knowledge they generate.

In summary, in an economy without spillovers, either sector-specific or intersectoral, market and planner allocate labor the same way, wages are the same and so are relative sector productivities. However, when the production function of each sector in the economy incorporates knowledge externalities in labor, and at least some of these externalities are positive, the presence of spillovers generates a wedge between the social and the private rates of return to labor. In other words, the existence of knowledge spillovers leads to a measurable market inefficiency.


3 Estimation Procedure

There has been some discussion in the literature as to how knowledge spills over among firms within the same industry or from one industry to another. The usual suspects are intermediate goods (supplier-driven spillovers), customer linkages, or direct transmission when productive processes (i.e. technology) are similar, even though products may be very different and the industries not transact with each other. Bernstein and Nadiri (1988) were among the first to note that industries that “borrow” other industries’ knowledge without transacting with them are usually industries where the rate of technological change is moderate to high. Recently, there has also been a flurry of new empirical studies that seem to equally support or debunk the evidence of localized spillovers derived from Marshallian agglomeration economies, particularly in Europe.10 Although the sectoral approach and level of aggregation used here preclude any specific conclusion on the effects of geographic concentration of production, or lack thereof, on spillovers, a matrix of estimated learning parameters ought to shed some light on this point.

In order to recover the learning parameters I will follow an estimation strategy that assumes constant returns to scale and perfect competition, and that stems directly from the relationship between rates of growth implied by the production function:

\[
\dot{Y}_i = \beta_{iK} \dot{K}_i + \beta_{iL} \dot{L}_i + \beta_{iM} \dot{M}_i + \sum_{j=1}^{n} \gamma_{ij} \dot{L}_j
\]

where \(\dot{Y}, \dot{K}, \dot{L},\) and \(\dot{M}\) are, respectively, the growth rates of the index quantities of output, physical capital, labor inputs, and intermediate inputs.

**Stage 1:** To perform the estimation of the learning parameters, I use the Tornqvist index of Total Factor Productivity (TFP), a discrete-time approximation to the Divisia index. TFP in each period is given by the difference between the growth rate

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10While Jaffe, Trajtenberg and Henderson (1993) appeared to prove the geographical nature of spillovers, Thompson and Fox-Kean (2005) have recently reassessed these findings, showing knowledge spillovers at the intranational level need not be geographically localized.
of output and the growth rate of all inputs, each weighted by its average cost-share:

$$TFP_i = \dot{Y}_i - \bar{S}_{iK}\dot{K}_i - \bar{S}_{iL}\dot{L}_i - \bar{S}_{iM}\dot{M}_i,$$

where $TFP_i$ is the growth rate of TFP for sector $i$ in terms of the differences of natural logarithms, and $\bar{S}_{iK}$, $\bar{S}_{iL}$, and $\bar{S}_{iM}$ are the average between-period shares of each input, also calculated using index prices and index quantities. Given that under perfect competition output elasticities are equal to factor shares, $\beta_{iK} = \bar{S}_{iK}$, $\beta_{iL} = \bar{S}_{iL}$, and $\beta_{iM} = \bar{S}_{iM}$, we can rewrite this equation as:

$$TFP_i = \sum_{j=1}^{n} \gamma_{ij}\dot{L}_j,$$

that is, the actual (observed) productivity growth equals the productivity growth predicted by the model’s production function. The variation of the residual associated to sector $i$ is the sum of each sector’s variation in employment weighted by sector $i$’s learning parameters (i.e. by what sector $i$ learns from each sector, including itself). It measures the costless gains to sector $i$ from the overall employment scheme. Hence, the residual is not a non-parametric estimation of a fixed parameter of the production function, but the reflection of a process.

**Stage 2:** To recover the learning parameters I minimize the distance between predicted and observed TFP growth, subject to values of the sum $q_i$ determined *ex-ante* and to a non-negativity constraint. For each period $t$ ($t \geq n$ guarantees a unique solution) I use annual growth rates computed from the five-year central moving averages of observed annual data. This eliminates or, at least, moderates the unwanted short-term effects of business cycles on the model’s productivity estimates (Bartelsman, Caballero and Lyons 1994), that will not reflect changes in the rate of utilization of inputs. The problem is then to choose parameters $\gamma_{i1}, \gamma_{i2}, ..., \gamma_{in}$, for
each sector $i = 1, 2, ..., n$, to

$$
\min \sum_{t=1}^{T} \left[ TFP_{it} - \sum_{j=1}^{n} \gamma_{ij} \dot{L}_{jt} \right]^2,
$$

subject to

$$
q_i = \sum_{j=1}^{n} \gamma_{ij} \quad \text{and} \quad \gamma_{ij} \geq 0, \forall i, j.
$$

The estimated coefficients are the learning parameters $\gamma_{ij}$ that solve this minimization problem. Each coefficient will measure how much productive knowledge flows within or between sectors, given the sectoral allocation of labor.

4 Data

The panel data set used in the estimation is an update on Dale W. Jorgenson’s original sectoral input-output database for the 1948-1979 period, also described in Jorgenson and Stiroh (2000), Jorgenson (1990), and Jorgenson et al. (1987). It covers the whole of the U.S. civilian economy from 1948 to 1991 and consists of annual observations on the value and the price of output and quality-adjusted inputs for 35 industries at roughly the 2-digit Standard Industrial Classification (SIC) level.

By using a data set that disentangles the quantity and quality effects of inputs, I ensure that the estimated TFP term will only capture the effects of costless spillovers, not of embodied technical change. For the same reason, estimates of knowledge spillovers are free from the upward aggregation bias associated with internal shifts in the composition of the inputs,\textsuperscript{11} and computed TFP growth becomes a lot smaller (Jorgenson and Griliches 1967). The labor series in the data correspond to hours worked adjusted for changes in their composition by age, sex, education, employment

\textsuperscript{11} In particular the compositional bias due to substitution towards assets with higher marginal products. E.g. a shift away from long-lived equipment in the capital stock, or the shift toward a more educated workforce. The shift toward IT, for example, increases the quality of capital, since computers, software, and communications equipment have relatively high marginal products.
class, and occupation. Growth in labor input reflects the increase in labor hours, as well as changes in the composition of hours worked as firms substitute among heterogeneous types of labor. Growth in labor quality is defined as the difference between the growth in labor input and hours worked. Likewise, the growth in capital quality is the difference between growth in capital services and capital stock.

In general, the use of quality-adjusted data allows us to distinguish between factor augmentation and TFP growth, which can then be safely attributed to factor augmentation (Jorgenson et al. 1994). In particular, accounting for the quality of the labor force is important as the majority of previous studies were not able to distinguish between marginal productivity (i.e. quality) improvements and spillovers proper.\footnote{According to Jorgenson and Stiroh (1994) and Jorgenson et al. (1994) about ten percent of the growth of the US economy in between 1947 and 1989 is due to increases in labor quality, which is the source of the spillover, but not the spillover itself.}

The sectoral TFP indexes measure the value-added output per combined unit of capital (K), labor (L), energy (E), and materials (M) in private business. The use of value added is more advantageous than gross output measures because industrial value added always sums up to total value added (GDP), independently of the degree of vertical and horizontal integration and of the proportion of intermediate goods used in production.\footnote{Aggregate value-added is immune to the kind of aggregation bias that occurs when sectoral share-weights change with the reallocation of GDP among sectors with different TFP levels and growth rates, creating a path dependence problem for the aggregate productivity index. Moreover, value added is impervious to outsourcing.} Intermediate inputs (energy plus materials) are treated symmetrically to capital and labor, thus taking into account substitution possibilities among all inputs.\footnote{Conceptually, TFP derived this way is closest to the producers’ approach.}

Note that it would not be necessary to assume constant returns to scale, because I use an independent measure of the return to capital to construct the share-weights in the estimates of sectoral TFP (Hulten 1973, 2001).
Grouping of Industries into Sectors

After eliminating government enterprises, I have aggregated the remaining 34 industries into six larger sectors, following the division drawn by Long and Plosser (1983) when analyzing real business cycles: Manufacturing (M), Mining (N), Construction (C), Services (S), Trade & Transportation (T), and Agriculture (A). The industrial composition of each sector can be seen in Appendix A1.

Aggregation of industries into a smaller number of sectors generates, on the one hand, an increase in the heterogeneity of the labor input, which the index number approach handles without difficulty and, on the other, an increase in the magnitude of sectoral spillovers $q_i$, as more sources of spillovers “pile up” in a given sector. A sector’s spillovers will reflect the combined effect of spillovers within the individual industries and the induced effects on those industries of intermediate inputs produced themselves with markups or externalities and under increasing returns that pile up in aggregation.15

Magnitude and Ranking of Sectoral Spillovers

Finally, the estimation requires an acceptable range for the value of knowledge spillovers received by each sector $q_i$. This range is set by empirical results in previous literature, where differences arise either from assumptions regarding inputs or from the level of aggregation applied. They are summarized as follows:

<table>
<thead>
<tr>
<th>Authors</th>
<th>Implied $q_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hall (1988, 1990), Domowitz, Hubbard and Petersen (1988), Caballero and Lyons (1992), Baxter and King (1991)</td>
<td>0.40 - 0.60</td>
</tr>
<tr>
<td>Morrison (1993), Bartelsman, Caballero &amp; Lyons (1991, 1994)</td>
<td>0.12 - 0.30</td>
</tr>
</tbody>
</table>

The earliest estimates imply that $q_i$ lies somewhere in between 0.4 and 0.6. However, these methodologies ignore the share of intermediate goods and, hence, produce

15 Basu and Fernald (1997) suggest most estimates of returns to scale suffer an upward aggregation bias whether the estimation uses gross or value-added output data.
estimates that are too large. The authors in the second group use aggregated gross output measures weighted to reflect the immediate suppliers or customers of the industry and obtain lower estimates. The large aggregation levels at which both groups work must also be taken into account.

The acceptable range for our \( q_i \) ought to be, then, closer to the second group’s estimates but start at a lower level, given that the model will be estimated for a six-sector economy. Once an acceptable range for the sectoral spillovers has been delimited, sectors are ranked \textit{ordinally} according to their learning potential, proxied by the proportion of labor employed in their R&D section, and assigned a corresponding level of \( q_i \):

<table>
<thead>
<tr>
<th>Ranking</th>
<th>Sector</th>
<th>( q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Manufacturing</td>
<td>M 0.30</td>
</tr>
<tr>
<td>2</td>
<td>Services</td>
<td>S 0.25</td>
</tr>
<tr>
<td>3</td>
<td>Mining</td>
<td>N 0.20</td>
</tr>
<tr>
<td>4</td>
<td>Trade &amp; Transportation</td>
<td>T 0.15</td>
</tr>
<tr>
<td>5</td>
<td>Construction</td>
<td>C 0.10</td>
</tr>
<tr>
<td>6</td>
<td>Agriculture</td>
<td>A 0.05</td>
</tr>
</tbody>
</table>

The interested reader can see Appendix A2 for a more thorough description of the data and method used in the ranking.

5 Results: A Matrix of Knowledge Flows

The estimated results for the learning parameters, expressed as percentages of the total sectoral spillover \( q_i \), are reported in Table 1, in the form of a \( 6 \times 6 \) matrix of intra- and inter-sectoral spillovers flowing from sectors of origin \( j \) to sectors of destination \( i \). The solid line encapsulates sectors into three larger divisions of the

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16 See Basu & Fernald (1995, 1997) for criticism to these earlier estimates.
17 Mansfield, Romeo, Schwartz, Teece, Wagner and Brach (1982) argue that firms use R&D for producing “inventions” and, besides, as a device to recruit and train people who eventually will move on to general management. Also, R&D not only transfers research findings, but often includes activities that are essentially technical service for parts of the firm, customers or suppliers.
economy: primary sector (agriculture), industry or secondary sector (manufacturing, mining, construction), and tertiary sector (trade & transportation, services).

Table 1 shows that manufacturing and trade & transportation are the main sources of spillovers for the economy, whereas services and agriculture do not generate any knowledge outflows. Most flows occur between industry and the tertiary sector, industry being the most dynamic both internally and externally, that is, industry as a whole generates and receives most flows in the matrix. All sectors receive spillovers from, at least, one other sector in the economy, but manufacturing and trade & transportation are the only sectors to learn from each other.

As for intra-sectoral flows, manufacturing is the one and only sector to learn from its own productive experience, above what it could learn from the rest of the economy as a whole. On the contrary, mining, construction, and trade & transportation are completely dependent on one single sector for the totality of their spillover. For both construction and trade & transportation this unique source is manufacturing.

In order to check the robustness of the above estimations, I have run a series of tests. To begin with, I subjected the estimation of the learning parameters to a common value $q_i = q \forall i$, and ran the estimation through the whole range of values of $q$. Second, I divided the whole period in two equal-length independent sub-periods and applied the original and the above constraints. Third, I increased the level of aggregation from six to three sectors and repeated the two previous procedures. Fourth, I imposed both types of constraints on $q_i$ but allowed the individual learning parameters to acquire positive, negative or zero values. And, fifth, I lifted all restrictions on both $q_i$ and the learning parameters. The results of all these previous tests failed to show any consistent pattern. Moreover, most estimates fell well beyond the range set by previous empirical studies. On the other hand, the value of the objective function is larger for the original, fully constrained case.

It is difficult to contrast the results of my estimation with previous results since the latter overwhelmingly refer to R&D generated spillovers. The closest reference would
be Bernstein (1988) estimates of spillovers generated by R&D capital (physical & human) and their private and social rates of return for seven Canadian two-digit SIC industries from 1978 through 1981. He finds that intra- and inter-sectoral spillovers do affect production costs and the structure of production. He also finds that spillovers create a wedge between the private and the social rates of return to the spillover-generating input. The first three columns of Table 2 show that his results and my estimations coincide broadly. Bernstein (1988) uses industries at a more disaggregate level, which contributes to explain why the value of his total sectoral spillovers is below 0.2. See Section 7 for comment on column four.

Van Stel and Nieuwenhuijsen (2004) use a completely different approach to determine whether inter-sectoral spillovers exist, and what their relative importance for achieving growth may be for different sectors of the economy. They assume geographical proximity to be a necessary condition for knowledge spillovers to occur. Using data for six macroeconomic sectors for 40 Dutch regions that cover the entire Netherlands from 1987 to 1995, they find that inter-sectoral knowledge spillovers are particularly important in the service sector (trade, transport & communication, financial services), whereas local competition is particularly important in the industry sectors (manufacturing and construction), which encourages an “innovation race.” One of the authors’ conclusions is that high extents of diversity encourage spillovers from industry sectors towards service sectors.

**Observed Transmission Channels**

In my model any industry employing skilled labor at any of its productive stages ought to benefit from labor-generated knowledge spillovers. However, the exact conduit for the transmission of knowledge is unspecified. In fact, the existing literature has not been conclusive either on the mechanism by which knowledge is transmitted, nor on the degree to which the transmission process is geographically localized. Knowledge as an input can be embodied in workers, in which case skills, experience and training
“travel” with workers moving within and among sectors. Alternatively, knowledge spillovers can be disembodied, so that they do not follow the transmission patterns of labor even if it is the generating input: they can be technology flows within and between industries and be proxied by measures of production and use of patents. In this section I address this issue by comparing the matrix of estimated knowledge flows with, successively, a matrix of worker flows and a matrix of technology or patent flows.

**Labor Flows**

A number of empirical papers find evidence of the diffusion of knowledge through labor mobility. However, most of these are case studies that refer to inter-firm mobility of highly skilled workers within the same sector, usually high-tech or R&D intensive industries, and usually within geographically localized industry clusters. To examine whether knowledge spillovers follow the pattern set by embodied knowledge flows, i.e. whether workers are the primary vehicle of knowledge spillovers when the whole economy is taken into account, I compare the matrix of learning coefficients in Table 1 with the equivalent matrix of average worker flows in Table 3.

To construct the transitional labor flow matrix in Table 3 I use a dataset provided by Maury Gittleman, of the Bureau of Labor Statistics, that consists of March to March matches of the Current Population Survey (CPS) from 1968 to 1992. The interested reader can see Appendix A3 for a more detailed description of data and procedure. Overwhelmingly, most of the turnover occurs within the same sector, for all sectors. Most sectors receive negligible inflows from manufacturing, services or trade, and the remaining sectors generate even smaller outflows.

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18 Note that knowledge embodied in workers only weans off productivity measures when, as is done here, changes in the marginal productivity of inputs are already measured into them.

19 For a rare inter-sectoral perspective, see Zucker, Darby and Brewer (1998) on the impact of highly skilled labor employed in the biotech industry on geographically close firms in the pharmaceutical, food processing, brewing, and agricultural industries. Moretti (2000, 2004a, 2004b) estimates the effect of highly educated workers on knowledge spillovers within a city, and the effect of knowledge spillovers on the education premium.
Technology Flows

Table 4 reports technology flows measured as the expenditure in R&D and patent production (origin) and the use of patents. It is constructed using data collected by Scherer (1984) on companies’ expenditures on R&D for the fiscal year 1974. Clearly, manufacturing is the main generator of technology inflows for all sectors, followed at a respectable distance by construction & services, which generate a small contribution to all sectors. No other sectors generate important outflows.

An updated version of Table 4 using data by Kortum (1995) on the number of U.S. and total patents applied for in the U.S. from 1957 to 1983, looks very much like Table 4.20

Comparison

Comparing Table 1 with Table 3 and Table 4, it is easy to see the matrix of relative learning parameter estimates is more similar to technology flows than worker flows. In order to obtain an exact measure of similarity or distance between matrices, I use three different metrics: rectilinear or Manhattan, Euclidean, and squared Euclidean distance, which penalizes large differences. By any of these, the matrix of learning parameters is closer to the matrix of technology flows than to the matrix of labor flows.

<table>
<thead>
<tr>
<th>Distance to the Matrix of Learning Parameters</th>
<th>Manhattan</th>
<th>Euclidean</th>
<th>Squared Euclidean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technology Flows</td>
<td>6.85</td>
<td>2.29</td>
<td>5.26</td>
</tr>
<tr>
<td>Labor Flows</td>
<td>10.36</td>
<td>2.84</td>
<td>8.06</td>
</tr>
</tbody>
</table>

While sectoral labor transitions proxy flows of embodied knowledge and occur mostly within sectors, information contained in patents and R&D is disembodied knowledge and travels mostly between sectors. As with the estimated matrix of knowledge flows in Table 1, manufacturing and the tertiary sector in Table 4 are the net

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20 Kortum (1995) uses the same 35-sector industrial classification, which makes it easier to update Table 4. I assume changes through time in the ranking of sectors of origin are mirrored by equivalent changes in sectors of use.
sources of knowledge in the economy. This would drive us to conclude that labor generated knowledge does not spill over through workers moving between sectors but, rather, that knowledge travels in a disembodied manner, proxied, to a certain extent, by technology flows as measured by the investment in and use of patents.

6 The Productivity Slowdown of the Early Seventies and the Shift in Spillovers

The productivity slowdown of the early seventies has been attributed to a large number of competing reasons. Explanations range from the reduction in real company financed R&D (Scherer 1984) to the incorrect measurement of output, particularly in services (Corrado and Slifman 1999). According to a different view (Greenwood and Yorukoglu 1997, Kortum 1997, Bessen 2002, Comin 2002), the productivity slowdown saw the underlying rate of technological change speed up. Alternatively, the slowdown has been explained as a consequence of the stagnation of the growth in the quality of human capital (Jorgenson et al. 1987, 1994) which, in this model, is equivalent to a reduction in sectoral knowledge spillovers.

The matrix of knowledge spillovers in Table 1 contributes to interpret the TFP residual for the entire period 1948-1991. To see whether the productivity slowdown of the early seventies is associated with changes in the creation or the absorption of knowledge spillovers, I split data into a pre-73 and a post-73 period, and do an independent estimation of the set of learning parameters for each. The resulting matrices are presented in Table 5.

Clearly, after 1973, only Trade & Transportation exhibits intra-sectoral knowledge flows. Moreover, the only knowledge flow received by Trade & Transportation is the one generated by itself. In contrast, Manufacturing and Construction, which prior to 1973 where the only ones to experience learning-by-doing, now obtain all their knowledge from Trade & Transportation, as does Mining. That is, before the
slowdown it was the industrial sector which generated most of the knowledge that benefitted the overall economy. But, after 1973, industry becomes completely dependent on Trade & Transportation, now the main generator of knowledge spillovers to the whole economy. After the slowdown, only Services benefits from knowledge generated by Manufacturing.

Thus, before the productivity slowdown manufacturing was the main source of knowledge for the whole economy. After 1973, trade & transportation takes over this role. Simultaneously, the number of and relative weight of sector-specific spillovers in the total diminish in favor of inter-sectoral knowledge flows.

7 Is the Market Efficient?

Clearly, the market does not allocate labor among sectors in an optimal or efficient manner; the wedge between the social and the private rates of return to labor, the spillover generating input, is reflected in the sectoral residuals via the knowledge spillovers (i.e. the learning parameters). This market inefficiency is reflected in Table 6 for the whole period and in Table 7 for the pre- and post-73 subperiods. These figures are best taken as an indication of the directionality and relative magnitude of market inefficiencies rather than in their absolute values.

It is important to note, once more, that in this model only the market’s allocation of labor can be improved upon, whereas the competitive allocation of capital and intermediates, the inputs that do not generate spillovers, is already optimal. Table 6 and Table 7 also present a description of the parameters that characterize the overall economy and each sector, that is, the parameters that are common to the market and optimal solutions for the whole period, and for the pre- and post-slowdown subperiods, respectively. These parameters are the following: consumers’ preferences for each sector, \( \alpha_i \); the output elasticities of each input by sector, \( \beta_{X_i} \), and for the overall economy, \( \beta_X \), \( X = K, L, M \); the total spillover each sector receives, \( q_i \); fixed ex-
ante, and the total spillover benefitting the economy, $Q$; the weight of intra-sectoral spillovers or learning-by-doing for each sector, $\gamma_{ii}/q_i$, and for the overall economy, $\sum \alpha_i \gamma_{ii}/Q$; the spillover generated for the benefit of the whole economy by each sector, $\Gamma_i$, and by the whole economy, $\Gamma$; the proportion of spillover generated by each sector specifically benefitting other sectors, $(\Gamma_i - \gamma_{ii})/(1 - \alpha_i)$, and its equivalent for the overall economy; and, finally, the relative sectoral allocation of the inputs that do not generate spillovers, $X_i/X$, where $X = K, M$.\(^{21}\)

For the whole period, manufacturing alone absorbs almost as much of consumers’ demand as the whole tertiary sector: 41% and 42%, respectively. Services is a distant second, with 24% of total demand, and trade & transportation is third, with 18%. Industry altogether absorbs more than half of total demand (52%). As for the intensity in the use of inputs, the overall U.S. economy between 1948 and 1991 spends half its resources retributing intermediate goods, destines 34% to paying for the labor it employs, and only 15% to capital used. Nevertheless, within the overall economy there are important sectoral differences. Whereas manufacturing, construction and agriculture show a high intensity in intermediates (65%, 55%, and 58%, respectively), above the overall economy, trade & transportation is the sector with the highest labor intensity, at 51%. Construction and services are also above the overall economy. As for capital, trade & transportation and agriculture exhibit the same intensity as the overall economy (15%), whereas mining and services are well above it (36% and 26%, respectively). It is worth noticing that factor intensities in manufacturing and agriculture are not so dissimilar, stressing the high level of industrialization of the agricultural sector in the U.S. Services and mining, on the other hand, exhibit intensities that are quite similar for all three factors of production.

As seen in Table 1, manufacturing is the only sector with learning-by-doing for the whole period. Therefore, the overall economy is also bound to benefit from intra-sectoral spillovers, although these will have less weight than for the manufacturing

\(^{21}\)The reader must keep in mind these figures refer to quality-adjusted inputs, not body or unit counts.
sector, specifically 30% of the total. By definition, whatever knowledge the economy generates is also the amount of knowledge the economy receives. Hence, the total spillover benefitting the economy is 0.23 and so is the total generated by the economy. By sector, manufacturing and trade & transportation generate 0.10 each for the whole economy, and mining 0.2. But when only knowledge generated for \textit{the rest} of the economy is taken into account, trade & transportation becomes the single main source of knowledge for sectors other than itself (0.12), whereas manufacturing falls second, at 0.06. This is due to consumers’ preferences. If a sector generates large spillovers to \textit{the rest} of the economy but the receiving sectors experience little demand, the final weighted spillover will be smaller. In contrast, a small spillover received by a high-demand sector will end up being larger. Differences in relative sectoral demand are also the reason why values for the knowledge received by \textit{the rest} of the economy may be higher than the total spillover generated.

Finally, in the last two rows we can see the weight each sector has in the demand for capital and intermediate goods relative to the total supply. These weights are the same for the planner and the market. Only the sectoral allocation of labor will differ. Manufacturing uses 53% of intermediates and 25% of capital. Services, by comparison, employs only 17% of intermediates, but 42% of capital, and trade & transportation, 12 and 18%, respectively. All other sectors’ weights are much smaller.

The pre- and post-slowdown descriptive parameters are not that different from those corresponding to the whole period. Note that consumers’ preferences for services exhibit a noticeable hike after 1973, from 21 to 30%, whereas preferences for manufacturing and, also, trade & transportation decrease. Comparing the last two rows of both sub-periods we can see the weight of manufacturing in the demand for both capital and intermediates decreases, while the importance of services increases noticeably, from 38 to 46% and from 13 to 22%, respectively. As for knowledge generated, after the slowdown trade & transportation clearly becomes the main generator of knowledge for the overall economy.
According to the model’s estimates of the competitive and optimal sectoral allocation of labor, output, and rates of return to labor for the whole period in Table 6, it would be optimal to increase the number of workers the market allocates to manufacturing by 32%, raising this sector’s output by 8%. Except for Mining, where employment would increase by 12%, and Services, where it would remain practically unchanged, it would be optimal for all other sectors to shed workers. Thus, Manufacturing’s share of total employment would go from 30% to 40%; wages in all sectors, except for Mining, would increase by, at least, 10%, with workers in Manufacturing perceiving wages 37% above market; and production economywide increasing by 1%.

If these results seem a bit excessive, they can be compared to those obtained by Bernstein (1988) and shown in the last column of Table 2. He also finds that spillovers create a wedge between the private and the social rates of return to the spillover-generating input. Moreover, his social-to-private ratio for the seven Canadian industries appears substantially higher than mine because a larger propensity to invest in R&D capital unambiguously leads to high intra-industry spillovers, which account for most of the differential between the social and private rates of return. Whereas here, investing in high-quality human capital (i.e. shifting the composition of the labor input toward higher marginal productivity workers) does lead to sector $i$ exhibiting larger intra-sectoral spillovers, but the upward impact on the social rate is dampened by the simultaneous downward pressure of higher cost-weighted shares of labor $\beta_{iL}$.

Table 7 presents the same estimates for the pre- and post-slowdown subperiods. After 1973 manufacturing losses weight in terms of labor employment in both the competitive and the optimal solution. In particular, if prior to the slowdown it would have been optimal to increase wage rates in manufacturing 20% over the market’s, after 1973 it is optimal to reduce them to three quarters of competitive wages. Mining is the only other sector for which it would be optimal to reduce wages. Services, on the other hand, experiences a surge of about ten percentage points in relative market
and optimal employment levels after 1973. Services is also the only sector for which
the optimal allocation of labor after the slowdown implies an increase in output,
while optimal wage rates remain 2% above the market’s. The largest wedge between
competitive and optimal wage rates happens in trade & transportation, where rates
ought to be 67% above the market’s.

Summarizing, the existence of costless productivity gains derived from labor-
generated knowledge spillovers creates a wedge between the market’s and the optimal
sectoral allocation of labor. Similarly, the market’s sectoral rates of return to labor,
the spillover-generating input, differ significantly from optimal rates.

8 Conclusions

In this paper I have estimated a model of labor generated knowledge spillovers. The
main purpose has been to shed some light on how knowledge diffuses across sectors
when one firm’s productive experience may enhance its own efficiency as much as
other firms’. That is, how productive knowledge transfers when there is learning-by-
observing as well as learning-by-doing, along the lines of Arrow (1962) and Jovanovic
and Rousseau (2002), in such a way that the economy benefits from intra- and inter-
sectoral knowledge spillovers.

After performing a two-stage measurement of spillovers and minimizing the dis-
tance between my model’s measurements and the observed data, I find that from 1948
to 1991 the Manufacturing sector was the undisputed engine of growth of the U.S.
economy, generating knowledge for itself and for the overall economy. However, the
productivity slowdown in the early seventies coincides with a change in the pattern
of generation and diffusion of spillovers. After 1973, trade takes over manufacturing
as the main generator of knowledge spillovers to the whole economy. Simultaneously,
the slowdown is associated with the decline of spillovers within sectors in favor of
spillovers between sectors; there seems to be some sort of sectoral integration.
As for the nature of knowledge and its transmission channel, I find that labor generated knowledge flows coincide with the patterns of other information flows, as the expenditure and use of patents and R&D, rather than with transitional labor flows. Although workers’ turnover happens within each sector, knowledge circulates increasingly between sectors. Hence, labor is not, after all, the channel through which labor-generated knowledge transfers.

Also, and in disagreement with some recent stream of literature concerning, especially, geographically localized spillovers, I find that there is indeed a wedge between how the market allocates and rewards labor and the optimal. That is, economy-wide, market wages do not absorb the totality of the costless productivity increases generated by the spillovers. Thus, the market allocates resources inefficiently: more resources should go to the main spillover generating sector, that is, manufacturing, so that employment in this sector ought to increase by 32%, and output by 8%; and wages in all sectors, except for Mining by, at least, 10%, with workers in Manufacturing perceiving wages 37% above market. Production economy wide would also increase by 1%.

These results are revealing about the feasibility of measuring knowledge spillovers and their effects on optimality in the allocation of resources by using a simple empirical framework. Throughout this paper, I have abstracted from dynamic effects arising from forward-looking decisions on investment in physical or human capital. The framework proposed here can be extended in those directions and evolve to a dynamic, Olley-Pakes type setup for estimating the production function. The results presented are thus encouraging about the feasibility of these extensions in future research.
Appendix

A1. Industry Classification

The following Table presents the equivalence between the original 35-sector classification used by D. Jorgenson, the SIC (1987), and the six-sector classification used in this article. Note sector 35, Government Enterprises, has been eliminated from the final selection.

<table>
<thead>
<tr>
<th>D. Jorgenson</th>
<th>SIC (1987)</th>
<th>6-sector Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture, fisheries and forestry</td>
<td>01, 02, 07, 08, 09</td>
<td>A</td>
</tr>
<tr>
<td>Metal mining</td>
<td>10</td>
<td>N</td>
</tr>
<tr>
<td>Coal mining</td>
<td>12</td>
<td>N</td>
</tr>
<tr>
<td>Oil and gas extraction</td>
<td>13</td>
<td>N</td>
</tr>
<tr>
<td>Non-metallic mining</td>
<td>14</td>
<td>N</td>
</tr>
<tr>
<td>Construction</td>
<td>15, 16, 17</td>
<td>C</td>
</tr>
<tr>
<td>Food and kindred products</td>
<td>20</td>
<td>M</td>
</tr>
<tr>
<td>Tobacco</td>
<td>21</td>
<td>M</td>
</tr>
<tr>
<td>Textile mill products</td>
<td>22 less 225</td>
<td>M</td>
</tr>
<tr>
<td>Apparel</td>
<td>23, 225</td>
<td>M</td>
</tr>
<tr>
<td>Lumber and wood</td>
<td>24 less 2451</td>
<td>M</td>
</tr>
<tr>
<td>Furniture and fixtures</td>
<td>25</td>
<td>M</td>
</tr>
<tr>
<td>Paper and allied</td>
<td>26</td>
<td>M</td>
</tr>
<tr>
<td>Printing, publishing and allied</td>
<td>27</td>
<td>M</td>
</tr>
<tr>
<td>Chemicals</td>
<td>28 less 282</td>
<td>M</td>
</tr>
<tr>
<td>Petroleum and coal products</td>
<td>29</td>
<td>M</td>
</tr>
<tr>
<td>Rubber and misc plastics</td>
<td>30, 282</td>
<td>M</td>
</tr>
<tr>
<td>Leather</td>
<td>31</td>
<td>M</td>
</tr>
<tr>
<td>Stone, clay, glass</td>
<td>32</td>
<td>M</td>
</tr>
<tr>
<td>Primary metal</td>
<td>33</td>
<td>M</td>
</tr>
<tr>
<td>Fabricated metal</td>
<td>34 less 348</td>
<td>M</td>
</tr>
<tr>
<td>Machinery, non-electrical</td>
<td>35</td>
<td>M</td>
</tr>
<tr>
<td>Electrical machinery</td>
<td>36</td>
<td>M</td>
</tr>
<tr>
<td>Motor vehicles</td>
<td>371</td>
<td>M</td>
</tr>
<tr>
<td>Transportation equipment &amp; ordnance</td>
<td>348, 2451, 37 less 371</td>
<td>M</td>
</tr>
<tr>
<td>Instruments</td>
<td>38</td>
<td>M</td>
</tr>
<tr>
<td>Miscellaneous manufacturing</td>
<td>39</td>
<td>M</td>
</tr>
<tr>
<td>Transportation</td>
<td>40 to 47 less 43</td>
<td>T</td>
</tr>
<tr>
<td>Communications</td>
<td>48</td>
<td>S</td>
</tr>
<tr>
<td>Electric utilities</td>
<td>491</td>
<td>S</td>
</tr>
<tr>
<td>Gas utilities</td>
<td>492</td>
<td>S</td>
</tr>
<tr>
<td>Trade (retail and wholesale)</td>
<td>50 to 59</td>
<td>T</td>
</tr>
<tr>
<td>Finance, insurance, and real estate (FIRE)</td>
<td>60 to 67</td>
<td>S</td>
</tr>
<tr>
<td>Services</td>
<td>70 to 89</td>
<td>S</td>
</tr>
<tr>
<td>Government enterprises</td>
<td>91 to 99, plus 43</td>
<td>-</td>
</tr>
</tbody>
</table>
A2. Ranking of Sectoral Spillovers $q_i$

I use the dataset by Hadlock, Hecker and Gannon (1991) to rank the paper’s six sectors by their capability to learn. Their data on R&D employment is derived from the Bureau of Labor Statistics Occupational Employment Statistics (OES) program, which provides current occupational employment data on salary and wage workers by industry.\(^{22}\) The data was collected in 1987, 1988, and 1989 for three-digit SIC industries, classified as high-tech if their proportion of R&D employment is at least equal to the average proportion for all industries. High-tech industries, in turn, are divided into R&D-intensive and R&D-moderate. An industry is R&D-intensive if its proportion of R&D employment is at least fifty percent higher than the average proportion for all industries surveyed. All other industries are R&D-moderate. All high-tech industries also show an above average annual pay level, the more so the higher the proportion of R&D employment in the industry.

This classification results in thirty R&D-intensive and ten R&D-moderate industries. All ten R&D-moderate industries and twenty-four of the R&D-intensive industries are in manufacturing, five are in services and one in mining (crude petroleum and natural gas operations).\(^ {23}\)


Only individuals with a strong attachment to the job market have been selected. Thus, only individuals that are white, male, aged 25 to 44 in the first year of the match, with 12 or more years of schooling, that have worked full-time year-round in both years of the match have been included in the sample. I have constructed average frequencies per transition using absolute frequencies and the relative weights of each transition with respect to the total number of observations per period or subperiod. That is:

$$W_n = \frac{L_{t+k,t+k+1}}{L_{t+1}},$$

and

$$f_{ij} = \sum_{n=t}^{t} f_{ij}(n) W_n,$$

\(^{22}\)Only manufacturing industries and selected non-manufacturing industries are surveyed for R&D employment, defined as the number of workers that spend the majority of their time in R&D, as determined by their employer.

\(^{23}\)The industry ranking according to percentage of R&D employment responds to what one would expect: in manufacturing, the top industries correspond to chemical manufacturing, missiles, space vehicles and parts, petroleum refining, computer and office equipment, and instruments (search and navigation, measuring and control devices, medical instruments and supplies, photographic equipment). The service R&D-intensive industries are research and testing, computer and data-processing, and then engineering and architectural services, miscellaneous, and management and public relations services.
where $i, j = A, C, M, N, S, T$, and $t$ and $\tau$ are, respectively, the first and the last “previous year” of every match or transition pair of years in a period. Therefore, $\sum_{n=1}^{\tau} W_n = 1$ for each period that spans from year $t$ (previous year) to year $\tau + 1$ (current year).

A4. Construction of the Technology Flow Matrix

I use Scherer (1984) data on companies’ expenditures on R&D for the fiscal year 1974 to construct a matrix of disembodied technology flows. The time span of the patent sample is the ten-month period from June 1976 through March 1977, because in the US, the average total lag between the invention (moment of R&D expenditure) and the issuance of a patent is assumed to be 28 months (9 months between the conception of an invention and the application for a patent, and 19 months between the application for a patent and its issue). The midpoint in the sample’s ten-month period is lagged exactly 28 months from June 30, 1974.

The sample comprises 15,112 patents or, roughly, 61 percent of all patents issued during the sample period to US industrial corporations. Following a verified and corrected version of the Federal Trade Commission’s Line of Business surveys, each patent is classified first by industry of origin, where the R&D expenditures have been recorded. Then these expenditures are carried over or transmitted to the industry(ies) of use via a fairly complicated algorithm. I aggregate Scherer’s ($41 \times 53$) matrix into a $(7 \times 6)$ version in Table 4.
References


### Table 1: Knowledge Spillovers, 1948-1991

<table>
<thead>
<tr>
<th>Sector of Destination (i)</th>
<th>Sector of Origin (j)</th>
<th>Spillover $q_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>N</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.55</td>
<td>0</td>
</tr>
<tr>
<td>Mining</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Construction</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>Services</td>
<td>0</td>
<td>0.30</td>
</tr>
<tr>
<td>Trade &amp; Transp.</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*a. All entries are percentages over sectoral spillover $q_i$.*

### Table 2: Bernstein’s Knowledge Spillovers for Canada, 1978-1981

<table>
<thead>
<tr>
<th>Sector of Destination (i)</th>
<th>Knowledge Spillovers</th>
<th>Social to Private Rates of Return $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Intra-sec. $\gamma_{ii}/q_i$</td>
<td>Inter-sec. $\sum \gamma_{ij}/q_i$</td>
</tr>
<tr>
<td>Chemical Products</td>
<td>0.84</td>
<td>0.16</td>
</tr>
<tr>
<td>Electrical Products</td>
<td>0.84</td>
<td>0.16</td>
</tr>
<tr>
<td>Aircraft &amp; Parts</td>
<td>0.81</td>
<td>0.19</td>
</tr>
<tr>
<td>Pulp &amp; Paper</td>
<td>0.81</td>
<td>0.19</td>
</tr>
<tr>
<td>Metal Fabricating</td>
<td>0.75</td>
<td>0.25</td>
</tr>
<tr>
<td>Food &amp; Beverage</td>
<td>0.77</td>
<td>0.23</td>
</tr>
<tr>
<td>Non-electrical Machinery</td>
<td>0.71</td>
<td>0.29</td>
</tr>
</tbody>
</table>

*a. $\omega^c = 0.1162$ for all industries.*
Table 3: Labor Turnover - Average Worker Flows, 1967-1991

<table>
<thead>
<tr>
<th>Sector of Inflow (i)</th>
<th>Sector of Outflow (j)</th>
<th>Total Inflow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>N</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.89</td>
<td>0</td>
</tr>
<tr>
<td>Mining</td>
<td>0.09</td>
<td>0.81</td>
</tr>
<tr>
<td>Construction</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>Services</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td>Trade &amp; Transp.</td>
<td>0.07</td>
<td>0</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.04</td>
<td>0</td>
</tr>
<tr>
<td>Unemployment</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Total Outflow 0.30 0.01 0.06 0.40 0.20 0.01 0.00

Table 4: Technology Flows - Production and Use of Patents, 1974

<table>
<thead>
<tr>
<th>Sector of Origin (j)</th>
<th>C &amp; Trade</th>
<th>Trans.&amp; Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>N</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>0.93</td>
<td>0</td>
</tr>
<tr>
<td>Mining</td>
<td>0.69</td>
<td>0.29</td>
</tr>
<tr>
<td>Construction</td>
<td>0.99</td>
<td>0</td>
</tr>
<tr>
<td>Services</td>
<td>0.94</td>
<td>0</td>
</tr>
<tr>
<td>Trade &amp; Transp.</td>
<td>0.94</td>
<td>0</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0.94</td>
<td>0</td>
</tr>
<tr>
<td>Final Consumption</td>
<td>0.96</td>
<td>0</td>
</tr>
</tbody>
</table>

Total Origin 0.96 0 0.02 0 0 0.01 100π

a. Mining as a sector of origin excludes petroleum and natural gas extraction. Construction & Services (including R&D); Trade, Finance & Real Estate (Trade & FIRE); Transportation & Public Utilities in the origin correspond, as a whole, to the sum of Services and Trade & Transportation in our sectors of use.
### Table 5: Pre- and Post-1973 Knowledge Spillovers (Pre-1973/Post-1973)

<table>
<thead>
<tr>
<th>Destination (i)</th>
<th>M</th>
<th>N</th>
<th>C</th>
<th>S</th>
<th>T</th>
<th>A</th>
<th>( q_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manufacturing</td>
<td>0.39/0</td>
<td>0/0</td>
<td>0.33/0</td>
<td>0.27/0</td>
<td>0/1</td>
<td>0/0</td>
<td>0.30</td>
</tr>
<tr>
<td>Mining</td>
<td>1/0</td>
<td>0/0</td>
<td>0/0</td>
<td>0/0</td>
<td>0/1</td>
<td>0/0</td>
<td>0.20</td>
</tr>
<tr>
<td>Construction</td>
<td>0.49/0</td>
<td>0/0</td>
<td>0/0</td>
<td>0/0</td>
<td>0/1</td>
<td>0/0</td>
<td>0.10</td>
</tr>
<tr>
<td>Services</td>
<td>0/0.67</td>
<td>0/0.33</td>
<td>0/0</td>
<td>0/0</td>
<td>1/0</td>
<td>0/0</td>
<td>0.25</td>
</tr>
<tr>
<td>Trade &amp; Transp.</td>
<td>0/0</td>
<td>0/0</td>
<td>1/0</td>
<td>0/0</td>
<td>0/1</td>
<td>0/0</td>
<td>0.15</td>
</tr>
<tr>
<td>Agriculture</td>
<td>0/0</td>
<td>0/0</td>
<td>0/1</td>
<td>1/0</td>
<td>0/0</td>
<td>0/0</td>
<td>0.05</td>
</tr>
</tbody>
</table>

\( a. \) All entries are percentages over total sectoral spillover \( q_i \).
### Table 6: Market and Optimal Solutions, 1948-1991

<table>
<thead>
<tr>
<th></th>
<th>M</th>
<th>N</th>
<th>C</th>
<th>S</th>
<th>T</th>
<th>A</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^s_i/L$</td>
<td>0.40</td>
<td>0.02</td>
<td>0.07</td>
<td>0.27</td>
<td>0.21</td>
<td>0.03</td>
<td>1.00</td>
</tr>
<tr>
<td>$L^c_i/L$</td>
<td>0.30</td>
<td>0.02</td>
<td>0.09</td>
<td>0.28</td>
<td>0.27</td>
<td>0.04</td>
<td>1.00</td>
</tr>
<tr>
<td>$L^s_i/L^c_i$</td>
<td>1.32</td>
<td>1.12</td>
<td>0.76</td>
<td>0.99</td>
<td>0.78</td>
<td>0.71</td>
<td>1.00</td>
</tr>
<tr>
<td>$Y^s_i/Y^c_i$</td>
<td>1.08</td>
<td>0.98</td>
<td>0.93</td>
<td>0.96</td>
<td>0.92</td>
<td>0.90</td>
<td>1.01</td>
</tr>
<tr>
<td>$w^s_i/w^c_i$</td>
<td>1.37</td>
<td>0.95</td>
<td>1.40</td>
<td>1.10</td>
<td>1.40</td>
<td>1.50</td>
<td>1.31</td>
</tr>
</tbody>
</table>

Descriptive parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>M</th>
<th>N</th>
<th>C</th>
<th>S</th>
<th>T</th>
<th>A</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_i$</td>
<td>0.41</td>
<td>0.03</td>
<td>0.08</td>
<td>0.24</td>
<td>0.18</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>$\beta_{Li}$</td>
<td>0.25</td>
<td>0.23</td>
<td>0.37</td>
<td>0.39</td>
<td>0.51</td>
<td>0.27</td>
<td>0.34</td>
</tr>
<tr>
<td>$\beta_{Ki}$</td>
<td>0.09</td>
<td>0.36</td>
<td>0.08</td>
<td>0.26</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
</tr>
<tr>
<td>$\beta_{Mi}$</td>
<td>0.65</td>
<td>0.41</td>
<td>0.55</td>
<td>0.34</td>
<td>0.34</td>
<td>0.58</td>
<td>0.50</td>
</tr>
<tr>
<td>$q_i$</td>
<td>0.30</td>
<td>0.20</td>
<td>0.10</td>
<td>0.25</td>
<td>0.15</td>
<td>0.05</td>
<td>0.23</td>
</tr>
<tr>
<td>$\gamma_{ii}/q_i$</td>
<td>0.55</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.30</td>
</tr>
<tr>
<td>$\Gamma_i$</td>
<td>0.10</td>
<td>0.02</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.00</td>
<td>0.23</td>
</tr>
<tr>
<td>$\Gamma_i - \alpha_i \gamma_{ii} / (1 - \gamma_{ii})$</td>
<td>0.06</td>
<td>0.02</td>
<td>0.00</td>
<td>0.12</td>
<td>0.00</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>$K_i/K$</td>
<td>0.25</td>
<td>0.06</td>
<td>0.04</td>
<td>0.42</td>
<td>0.18</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>$M_i/M$</td>
<td>0.53</td>
<td>0.02</td>
<td>0.09</td>
<td>0.17</td>
<td>0.12</td>
<td>0.06</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 7: Pre- and Post-1973 Market and Optimal Solution

<table>
<thead>
<tr>
<th>Description</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
<th>Pre</th>
<th>Post</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L^s_i/L$</td>
<td>0.42</td>
<td>0.36</td>
<td>0.02</td>
<td>0.02</td>
<td>0.07</td>
<td>0.07</td>
<td>0.23</td>
<td>0.34</td>
<td>0.22</td>
<td>0.19</td>
<td>0.04</td>
<td>0.02</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$L^c_i/L$</td>
<td>0.33</td>
<td>0.27</td>
<td>0.02</td>
<td>0.02</td>
<td>0.09</td>
<td>0.09</td>
<td>0.23</td>
<td>0.35</td>
<td>0.29</td>
<td>0.25</td>
<td>0.05</td>
<td>0.03</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$L^s_i/L^c_i$</td>
<td>1.31</td>
<td>1.33</td>
<td>1.11</td>
<td>1.14</td>
<td>0.78</td>
<td>0.75</td>
<td>1.00</td>
<td>0.97</td>
<td>0.77</td>
<td>0.79</td>
<td>0.71</td>
<td>0.72</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$Y^s_i/Y^c_i$</td>
<td>1.08</td>
<td>1.00</td>
<td>1.08</td>
<td>0.98</td>
<td>0.91</td>
<td>0.87</td>
<td>0.99</td>
<td>1.05</td>
<td>0.84</td>
<td>0.86</td>
<td>0.91</td>
<td>0.91</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>$w^s_i/w^c_i$</td>
<td>1.20</td>
<td>0.75</td>
<td>1.00</td>
<td>0.94</td>
<td>1.64</td>
<td>1.44</td>
<td>1.08</td>
<td>1.02</td>
<td>1.39</td>
<td>1.67</td>
<td>1.56</td>
<td>1.50</td>
<td>1.27</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Descriptive parameters

$$\alpha_i$$ | 0.44 | 0.38 | 0.02 | 0.03 | 0.09 | 0.08 | 0.21 | 0.30 | 0.19 | 0.18 | 0.06 | 0.04 | 1.00 | 1.00 |
| $\beta_i$ | 0.26 | 0.24 | 0.24 | 0.22 | 0.36 | 0.39 | 0.39 | 0.39 | 0.54 | 0.47 | 0.29 | 0.24 | 0.35 | 0.34 |
| $\beta_i$ | 0.10 | 0.09 | 0.36 | 0.36 | 0.07 | 0.08 | 0.29 | 0.23 | 0.16 | 0.13 | 0.14 | 0.17 | 0.16 | 0.15 |
| $\beta_i$ | 0.64 | 0.67 | 0.40 | 0.42 | 0.57 | 0.53 | 0.32 | 0.38 | 0.30 | 0.39 | 0.57 | 0.59 | 0.50 | 0.51 |
| $q_i$ | 0.30 | 0.30 | 0.20 | 0.20 | 0.10 | 0.10 | 0.25 | 0.25 | 0.15 | 0.15 | 0.05 | 0.05 | 0.23 | 0.23 |
| $\gamma_i/q_i$ | 0.39 | 0.00 | 0.00 | 0.00 | 0.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.10 | 0.00 | 0.00 | 0.25 |
| $\Gamma_i$ | 0.06 | 0.05 | 0.00 | 0.02 | 0.08 | 0.00 | 0.04 | 0.00 | 0.05 | 0.15 | 0.00 | 0.00 | 0.23 | 0.23 |
| $\Gamma_i-\alpha_i\gamma_i$ | 0.01 | 0.08 | 0.00 | 0.02 | 0.08 | 0.00 | 0.05 | 0.00 | 0.06 | 0.15 | 0.00 | 0.00 | 0.17 | 0.20 |
| $K_i/K$ | 0.28 | 0.23 | 0.06 | 0.07 | 0.04 | 0.04 | 0.38 | 0.46 | 0.19 | 0.16 | 0.05 | 0.04 | 1.00 | 1.00 |
| $M_i/M$ | 0.56 | 0.49 | 0.02 | 0.02 | 0.10 | 0.08 | 0.13 | 0.22 | 0.11 | 0.14 | 0.07 | 0.05 | 1.00 | 1.00 |
\[ \gamma_{12} = \gamma_{21} > 0 \]
\[ \gamma_{12} = \gamma_{21} = 0 \]
\[ \gamma_{12} > 0, \gamma_{21} = 0 \]

Figure 1: Market and Optimal Allocation of Labor \( (n = 2, \beta_{1L} = \beta_{2L}, \text{and } q_1 > q_2) \)