Why factor income shares seem to be constant?

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Abstract

The common assumptions that labor income share does not change over time and that factor income shares are equal to the elasticity of output with respect to factors have had important implications for economic theory. However, there are various theoretical reasons why the elasticity of output with respect to reproducible factors should be correlated with the stage of development. In particular, the behavior of international trade and capital flows and the existence of factor saving innovations imply such a correlation. If this correlation exists and if factor income shares are equal to the elasticity of output with respect to factor, the labor income share must be negatively correlated with the stage of development. We propose two complementary explanations for why labor income share has no trend: (i) the existence of a labor intensive sector which produces non tradable goods; and (ii) the existence of more than two factors. It is possible that an increasing trend in physical capital income share is compensated by a decreasing trend in land income shares. Similarly, an increasing trend in human capital income share may be compensated by a decreasing trend in raw labor income share.

JEL classification: E1, F0, O0.

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I. Introduction

The works by Cobb and Douglas (1928) and Kaldor (1961) created a paradigm for macroeconomics. The idea that labor income share does not decrease or increase with time or with the stage of development[1] had important implications in macroeconomics and growth theory. Considering an aggregate production function, if factor income shares are constant and the price of each factor is determined by its marginal productivity then the elasticity of output with respect to each factor is also constant. In other words the constancy of factor shares implies that the Cobb-Douglas is a good approximation for the aggregate production function.

Subscribing to this paradigm, almost all of the literature on economic growth accounting assumes that the elasticity of output with respect to capital (and labor) is constant and have concluded that the major part of economic growth is not explained by factor accumulation but by growth in TFP

[1] Recently the result was confirmed by Gollin (2002) in a serious empirical work.

(see Denison, 1967; Easterly and Levine, 2002; Solow, 1957 or Young, 1994 among others). However there are 3 theoretical reasons why the elasticity of output with respect to reproducible factors, namely, physical capital and human capital, should be positively correlated with the stage of development:

1. International Trade. Eli Hecksher (1919) and Bertil Ohlin (1939) argue that comparative advantage arises from differences in national factor endowments. The Hecksher-Ohlin theory predicts that countries specialize and export those goods that make intensive use of locally abundant factors, while importing (producing less) goods that make intensive use of factors that are locally scarce. In other words, countries specialize in goods characterized by a high elasticity of output with respect to locally abundant factors. Therefore, international trade and specialization should increase the elasticity of aggregate output with respect to abundant factors and decrease the elasticity of aggregate output with respect to scarce factors. Finally, if factor income shares are equal to the elasticity of aggregate output with respect to factors then international trade and specialization should increase the income share of abundant factors and decrease the income share of scarce factors.

2. Foreign Direct Investment (FDI). According to John Dunning (1988) location-specific advantages can help explain the nature and direction of FDI. By location specific advantages, Dunning means the advantages that arise from using resource and factor endowments that are tied to a particular location. Therefore, firms producing labor-intensive goods are likely to invest in labor abundant countries while firms producing capital-intensive goods are likely to invest in capital abundant countries. This implies that the behavior of FDI generates, in each country, an expansion in the sectors that use intensively abundant factors and, as a result, should increase the elasticity of aggregate output with respect to abundant factors and the income share of the abundant factors.

3. Factor Saving Innovations. Economic growth models of biased innovations have been proposed by Kennedy (1964), Zeira (1998), Acemoglu (2002), Boldrin and Levine (2002) and Zuleta (2003) among others. In these models factor scarcity generates incentives to invest in factor saving innovations, that is, people invest to reduce the need of scarce factors and increase the relative use of abundant factors. If factor prices are determined by marginal productivity of factors then laborsaving innovations reduce the income share of workers and increase capital

income share. In more general terms, the income share of non-reproducible factors decreases with the stage of development while the income share of reproducible factors increases (Zuleta, 2003).

If factor prices are determined by their marginal productivity and, consequently, the income share of each factor is identical to the elasticity of output with respect to such a factor, then the three theoretical arguments described above imply that the income share of reproducible factors should be positively correlated with the stage of development. So how can we explain the absence of a trend for capital income share? In the following lines we propose two complementary explanations:

1. Labor Intensive Sector.

In economic growth, theorists are used to working with aggregate production functions. However, different sectors are likely to have different production functions and reveal different behaviors. In particular, consider a very labor intensive non-tradable sector where the possibilities of labor-saving innovations are low, namely, services, and suppose that this sector produces a normal good (X). The other sector produces the good Y, which is tradable and uses capital in a more intensive way. Under these circumstances, as an economy grows the demand for the good X also grows, which creates the need to hire more workers. As capital grows the cost of labor also increases, and so does the relative price of good X.

If the effect of the increase in prices (in sector X) on labor demand is higher than the effect of the increase in capital (Y), then, as the economy grows, more labor is allocated to the production of good X. Under such circumstances labor income share decreases as the stock of capital grows. As it will become clear, this happens when the elasticity of substitution between goods X and Y is low enough. If this is the case, the question to ask is why capital income share does not decrease with the stage of development. The answer to this question can be found in the International Trade, Foreign Direct Investment and Factor Saving Innovations theories.

2. More than 2 factors.

The standard measures of labor income share includes raw labor and human capital income shares. In the same way, the standard measure of capital income share includes land income share. Therefore, it is possible that an increasing trend in physical capital income share is

compensated by a decreasing trend in land income share. Similarly, an increasing trend in human capital income share may be compensated by a decreasing trend in raw labor income share.

The rest of this paper is organized as follows. In Section 2 we present a two-sector model where labor income share increases with the stage of development if aggregate technology is constant. In Section 3 we present an empirical exercise; using the standard growth regression methodology, we take into account the fact that factor shares may depend on factor abundance and verify if there exists a constant relation between elasticity of output with respect to reproducible factors and income per capita. Section 4 concludes.

II. Two Sector Model

Consider a two-sector model, where each sector produces a different good. Both goods *X* and *Y* are normal goods and their production functions are as follows:

$$(1)\ Y = K^{a} L_{y}^{1-a} \quad X = BL_{x}$$

where *K* is capital, *L* is labor, the sub-indexes *x* and *y* account for the amount of labor devoted to the production of goods *X* and *Y* respectively, *B* is the labor productivity in the production of the good *X*, a is the elasticity of output with respect to capital in the production of the good *Y,* and $1-a$ is the elasticity of output with respect to labor in the production of the good *Y.*

For simplicity we assume constant labor supply and normalize $L=1$. Therefore, $L_y=1-L_x$. We also call *k* the capital labor ratio in sector *Y*, namely, $k = \frac{K}{1-Lx}$; use good *Y* as a numeraire, and call *p* the relative price of good *X*. Markets are competitive so wage (w) is determined by the marginal productivity of labor, and the interest rate (r) is determined by the marginal productivity of capital, namely,

$$(2)\ w = (1-a)k^{a} = pB \text{ and } r = ak^{a-1}.$$

Given that both goods are normal we can assume a CES utility function,

$$U = \left(Y^{f} + X^{f}\right)^{\frac{1}{f}}$$

where the elasticity of substitution between goods X and Y is given by, $e_{X,Y} = \frac{1}{1-f}$. Therefore, if the elasticity of substitution is positive, then $\phi \leq 1$, and if it is lower than one then $\phi \leq 0$. If agents are utility maximizers then relative prices and consumption should relate in the following way:

$$p = \left(\frac{C_Y}{C_X} \right)^{1-f}$$

where *Cy* and *Cx* are the consumption of good *Y* and *X* respectively. Therefore if the relative price of good *X* grows then consumers substitute consumption of good *X* for consumption of good *Y*. For simplicity let me assume that total demand behaves in the same way that consumption does, that is, the demand for investment (*I*) of goods *X* and *Y* is such that $p = \left(\frac{I_Y}{I_X} \right)^{1-f}$. In this case,

$$(4)\ p = \left(\frac{Y}{X} \right)^{1-f} = \left(\frac{K^a (1-L)^{1-a}}{BL} \right)^{1-f}$$

Using equation 2 and rearranging yields

$$(4')\ p = \left(\frac{1}{1-a} \frac{(1-a)^{\frac{1}{a}} K}{pB - (1-a)^{\frac{1}{a}} K} \right)^{\frac{1-f}{f}}$$

From equation 4′ it follows that in the absence of labor saving innovations (constant α) the price of good *X* is positively correlated with the stock of capital *K*. Combining equations 4 and 2 we get:

$$(5)\ K = \left(\frac{B}{(1-a)} \right)^{\frac{1}{af}} (1-L_x)^{\frac{1-f(1-a)}{af}} (L_x)^{\frac{f-1}{af}}.$$

From equation 5, if $f > \frac{2}{2-a}$ or $f < 0$ then $\frac{\partial K}{\partial L_x} > 0$. In other words, if the substitutability between *X* and *Y* is low enough, namely, $e_{X,Y} < \left(\frac{2-a}{a} \right)$, then as the economy accumulates capital, the share of workers devoting their time to the production of good *X* grows[2]. Therefore: (i) given the elasticity of output with respect to capital (a), for low elasticity of substitution between capital and labor there exists a positive relation between capital and share of labor in the production of good *X*. Moreover, if $e_{X,Y} < 1$ then $\frac{\partial K}{\partial L_x} > 0$ for any $a < 1$. And (ii) given the elasticity of

[2] If $0 < f < \frac{2}{2-a}$ then $1 < e$ and $\frac{2-a}{a} < e$

substitution between capital and labor, for low levels of elasticity of output with respect to capital (a) there exists a positive relation between capital and share of labor in the production of good X, that is, $\frac{\partial K}{\partial L_x} > 0$ [3]. Therefore, for reasonable values of $e_{X,Y}$ and a there exists a positive relation between the stock of capital and the share of workers in the production of good X[4].

Now, the labor income share of the economy (*LISh*) is given by the ratio between wage (*pB*) multiplied by the number of workers (*L=1*) and total output ($AK^a L_y^{1-a} + pBL_x$):

$$LISh = \frac{pB}{K^a (1 - L_x)^{1-a} + pBL_x}$$

Using equation 2 and rearranging,

$$(6)\ LISh = \frac{1-a}{2 - L_x(1+a)}$$

From equation 6 it follows that if the elasticity of output with respect to capital (α) is constant then labor income share is positively correlated with the share of workers devoted to the production of the good *X*.

Finally, from equations 5 and 6 it follows that, if $e_{X,Y} < \frac{2-a}{a}$, then labor income share grows as K grows, that is, as the economies grow labor income share grows. This result has the following implications: (i) if $e_{X,Y} < \frac{2-a}{a}$ and the elasticity of output with respect to capital is constant, then as the economy grows labor income share grows; and (ii) if the elasticity of output with respect to capital is low, then as the economy grows, labor income share grows.

From table 1, it is evident that for all the countries in the sample there exists a positive trend in the share of workers devoted to the production of services. Therefore, if the elasticity of output with respect to capital is constant, then labor income share should have a positive trend.

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[3] Note that if $\varepsilon=2$ then $a<0.67$ implies $\frac{\partial K}{\partial L_x} > 0$, and if $a=0.4$ then $\varepsilon<4$ implies $\frac{\partial K}{\partial L_x} > 0$.

[4] Estimates of Armington elasticities, that is, the elasticity of substitution between home and imported goods, have been supplied by a variety of scholars (see Blonigen and Wilson, 1999, for example). In these studies it is found that the Armington elasticities generally fall below 1.5. Given this result it is likely that the elasticity of substitution between tradable and non tradable goods is below 1.5.

Table 1

Employment in Services as a percentage of civilian employment								
	1965	1970	1975	1980	1985	1990	1995	2000
Italy	36.8	40.3	44.1	47.8	55.2	58.5	59.2	62.2
Spain	-	-	39.7	44.9	50.1	54.9	61	62.3
Australia	38	40.5	46.6	49.3	52.9	55.3	60.3	63.6
Japan	44.7	46.9	51.5	54.2	56.4	58.7	60.7	63.7
Germany	40.7	42.9	47.6	51	53	57.3	60.5	63.9
Finland	-	-	-	51.8	56.5	60.7	64.5	66.3
Switzerland	41.2	45.4	-	55	58.3	63.6	66.9	69.8
France	43.1	47.2	51.1	55.5	60.8	64.6	69	72
Sweden	-	-	-	62.1	66.1	67.8	71	72.9
Austria	53.1	55	59.4	62.5	66.3	69.2	72.1	73.1
United Kingdom	49.5	52	-	59.8	65.8	68.7	70.1	73.2
Norway	-	-	56.1	61.6	65.4	68.8	71.4	73.8
Canada	-	62.6	64.3	66.9	70.5	71.7	74	74.1
USA	-	61.1	65.4	65.9	68.8	70.9	73.1	74.4
Source: OECD								

III. Physical and Human Capital

To test the hypothesis that increases in physical capital (human capital) income share is compensated by decreases in land (raw labor) income share, we proceed as follows: First, in panel data regression we estimate the relevant parameters related to the elasticity of output with respect to land, physical capital and human capital. Second, by using the estimated parameters, we calculate the elasticity of output with respect to the factors.

The existing literature of empirical economic growth has two main branches. Authors like Barro and Sala-i-Martin (1992) and Mankiw, Romer and Weil (1992) assume that factor shares are equal across countries. Others, such as Durlauf and Johnson (1995) allow for differences in factor shares between different groups of countries but assume that factor shares are the same within groups. We need to go one step ahead and allow for differences in factor shares not only among groups, but also among countries within groups. To do so, we use the standard growth regression methodology taking into account the fact that factor shares may depend on factor abundance.

To test the model, we use the data of De la Fuente and Domenech (2001) from OECD[5] countries for GDP, physical capital and human capital. For the variables land (*T*) and raw labor (*L*) we use data from the World Development Indicators.

Consider an aggregate production function, which combines two reproducible factors, human and physical capital (*H* and *K*) and two non-reproducible factors, land and raw labor (*T* and *L*) in a Cobb-Douglas: $Y = AK^{a} H^{b} T^{g} L^{1-a-b-l}$. Therefore, income per capital is $y = Ak^{a} h^{b} t^{g}$, where *k* is capital per worker, *h* is human capital per worker and *t* is land per worker. By taking logarithms and differences between period zero and period one, we obtain:

$$\log(y_1) - \log(y_0) = \log(A_1) - \log(A_0) + a_1 \log(k_1) - a_0 \log(k_0) + b_1 \log(h_1) - b_0 \log(h_0) + g_1 \log(t_1) - g_0 \log(t_0)$$

We assume that the elasticity of output with respect to the different factors can be different at the beginning of different periods. However, since we are considering periods of five years we assume that the effect of the increments in the elasticities are negligible. Therefore, the growth rate of income per capita is given by:

$$\frac{\Delta y}{y} = \frac{\Delta A}{A} + a_0 \frac{\Delta k}{k} + b_0 \frac{\Delta h}{h} + g_0 \frac{\Delta t}{t}$$

For the growth rate of total factor productivity we test for technological convergence. For each country *i*, at any time, TFP can be obtained as $A_i = \frac{y_i}{t^{g_i} k_i^{a_i} h_i^{a_i}}$. Therefore, if there is technological convergence, then

$$\frac{\Delta A_i}{A_i} = m + d\left(\log y_{0,i} - a_{0,i} \log k_{0,i} - b_{0,i} \log h_{0,i} - g_{0,i} \log t_{0,i}\right) + e_i$$

where the parameter d indicates the speed of technological convergence, ε is a change in technology that is not explained by technological convergence and is country specific and e_i is partially captured by country and time dummies.

[5] Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, USA, UK. Years: 1960, 1965, 1970, 1975, 1980, 1985, 1990, 1995.

The changes in a are reflected in a reduction of g, so $g = r - a$, where r is a parameter to be estimated. For the functions a and ß we use the functions $a = h(1 + log(k)/(log(k)+1))$ and $b = l(1 + log(h)/(log(h)+1))$, respectively[6]. In this way, the elasticity of output with respect to reproducible factors has two components, one is fixed, and the other one is variable and bounded from above. Finally, we use two different measures for the variable *T:* the first is the total area of the country measured in hectares, and the second is the arable land measured in hectares.

Table 2 presents the results. The coefficients g, l and r are positive and significantly different from zero in both cases. Thus evidence supports the idea that the contribution of physical and human capital accumulation to economic growth is higher in capital abundant economies. Additionally, $a+\gamma$ is 0.35 in one case and 0.36 in the other. These numbers are similar to the conventional wisdom which estimates capital income share around 1/3.

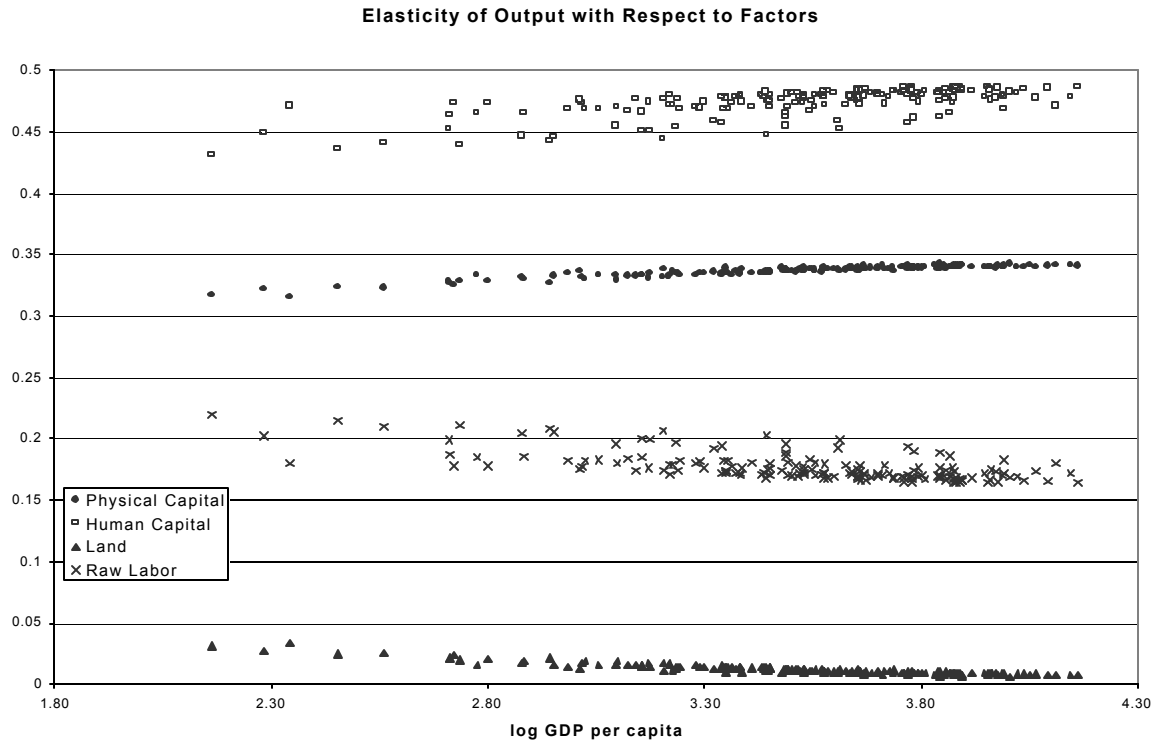
Table 2

	1	2
	Total Area	Arable Land
η	0.198	0.186
	(0.000)	(0.000)
λ	0.325	0.377
	(0.000)	(0.000)
ρ	0.360	0.349
	(0.000)	(0.000)
δ	0.074	0.092
	(0.000)	(0.000)
μ	0.094	0.133
	(0.000)	(0.000)
Adjusted R-square	0.789	0.650
Number of Observations	123	100

Country Dummies: Belgium, USA, Canada, France, Italy, Netherlands, and Spain.
Time Dummies: 1970, 1975, and 1980
Multiplicative Dummy for change in physical capital: Ireland
Multiplicative Dummy for change in human capital: Finland, Ireland and New Zealand.

[6] Other specifications were considered and the results didn't change.

Finally, we use the data and the estimated parameters to calculate the values of a, b and r for all the countries in different years. Figure 1 shows that, according to our, results there exists a positive correlation between the elasticity of output with respect to reproducible factors and the level of output per worker.



IV. Conclusions

International Trade, Foreign Direct Investment and Factor Saving Innovations theories provide three different theoretical reasons why the elasticity of output with respect to reproducible factors, namely, physical capital and human capital should be positively correlated with the stage of development. Similarly, if factor prices are determined by factor marginal productivities then the elasticity of output with respect to factors determines factor income shares. This implies that there are 3 different theoretical reasons why the income share of reproducible factors should be positively correlated with the stage of development

In this paper we present two reasons why labor income share can be constant even if factor prices are determined by factor marginal productivities and the elasticity of output with respect to reproducible factors increases with the stage of development. The first reason is the existence of a

labor-intensive sector that is not subject to labor saving innovations. The second reason is the existence of more than one reproducible factor and more than one non-reproducible factor.
The main point of the paper is that, even though labor income share does not present any trend, the elasticity of output with respect to reproducible factors depends on the stage of development of the countries. This result has important implications for empirical exercises related to economic growth (accounting and econometrics). In particular, the conclusion that the major part of economic growth is not explained by factor accumulation but by growth in TFP needs to be revised. Similarly, the endogeneity of the elasticity of output with respect to reproducible factors has implications for fiscal policy.

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