Choosing ‘Me’ and ‘My Friends’
Identity in a Non-Cooperative Network Formation Game with Cost Sharing

Working Paper*

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Abstract

This paper introduces the choice of identity characteristics, and, commitments to these characteristics, in a network formation model where links costs are shared. Players want to link to the largest group given that linking costs are decreasing (increasing) in commitments for same (different) identity. We study conditions under which these choices allow for networks with multiple identities. We find that whether the choice of identity itself gives any utility or not, there will be Nash networks featuring multiple identities. Moreover, if the choice of identity directly adds utility, networks with multiple identities will be efficient and survive the dynamic process.

Keywords: Identity, Network Formation, Cost Sharing Links

JEL Classification: D85, Z13, C72

*I would like to thank conference participants at ‘LSU Conference on Networks: Theory and Applications’ for excellent comments and suggestions.

1 Introduction

It has long been acknowledged in the social sciences that identity is fluid and it is a conscious choice. Here the word "identity" can be decomposed into two sets of ideas - one, as the set of characteristics of an individual, and two, as the degree of importance he attaches to each characteristic in that set. In his seminal paper, Horowitz (1977) cites many examples of smaller identity groups assimilating to form a larger identity group; as well as the opposite examples of differentiation, where smaller identity groups emerge from what used to be a larger cohesive identity group. These changes in identity are clearly a result of the conscious choices of these people and they clearly show that identity has an element of fluidity to it; that it is constantly evolving. [1] An interesting example of demographically similar societies choosing different commitments to their identity is to be found in Croatia where "In the years 1991 and 1992 the war between Serbs and Croats was raging through Croatia, but the ethnically mixed region of Gorski kotar (located east of the Adriatic seaport of Rijeka) managed to escape an armed conflict. The inhabitants there, Croats and Serbs alike, overcame national tensions and tried hard to preserve an "active peace"." [2]

The question of interest then is, what economic motivation drives us to pick our respective identities? We can think of identities evolving as a result of players choosing their networks and identity at the same time. The networks are needed for the transfer of information, for the formation of bargaining units, for insurance, etc. Identity serves as an adhesive to the formation of these economically useful networks - the link needed to connect a pair is expensive, but by choosing similar identities, these costs can be lowered.

The first contribution of this paper is in the introduction of choice of identity. In the network formation models, players choose to form links and their benefits are increasing in the number of other players they are connected to (directly or indirectly) in the final network configuration. A player would like to belong to the largest network possible and form the least possible links. We change the standard network formation game by allowing players to simultaneously choose their identities as well as their links, where, the choice of identity will have repercussions for the network profits. We define 'identity' along a single dimension and each player is assigned a single characteristic along that dimension. The player could have this characteristic exogenously given (or not), but he will always choose his "commitment" to that identity characteristic. The choice of commitment for each player is captured by the variable $\theta \in [0, 1]$, where a higher number denotes a stronger commitment to the characteristic. Identity and the commitment to identity then have an impact on the cost of links; individuals with same characteristic will find it cheaper to link as each increases his commitment, whereas, individuals with different characteristics will find it costlier to link as each increases his commitment.

The other contribution of this paper to the literature on network formation, lies in the fact that in this paper, links will be formed based on offers of contribution from both players. As seen in Figure 1, both players will make an offer of how much of the link cost they are willing to bear, if the offers of the two

[1] Another good example is to be found in Chandra (2001) she cites the example of the Hindu-Muslim divide in India in 1989 giving way to a divide along caste lines by 1990.

[2] $http://www.cis.or.at/projects/gorski\_kotar.html$

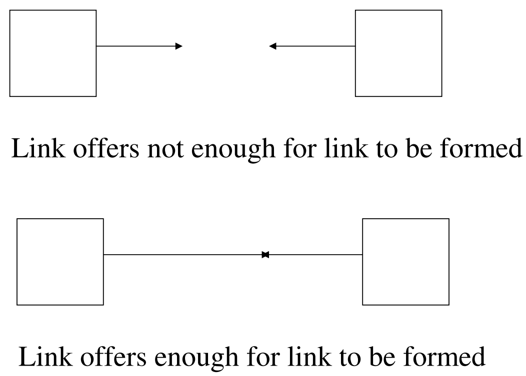

Figure 1: Link Cost Sharing

players add up to be enough to form the link, the link is formed, otherwise not. If the link is formed, players bear a share of the cost as per their initial offer. The cost of the link depends on the choice of identity and commitment as mentioned above. Each player's link strategy is to choose how much to offer for each of his possible links. The resulting network depends on the links actually made. Benefits of belonging to the network are increasing in the set of players linked to, directly or indirectly. We also make the assumptions that adding a profitable link is profitable, a link in a profitable strategy is profitable on its own and that player labels are irrelevant. We study the level of fragmentation by identity in the Nash equilibrium of this game. We also look at a dynamic version of the game, where a player is chosen each period, and this player chooses his identity, commitment level and also the link offers he would like to make. In the same period, the players to whom link offers are made, respond by choosing their identity, commitment level and also whether to accept the link offer of the initiating player.

In the first part of the paper, we fix the identity characteristics exogenously. Hence, a player's strategy consists of choosing his commitment level and his link strategy. The resulting Nash network of the static game will either be empty, separated by identity, or, all players will be connected. In other words, with identity characteristics fixed, it is likely that the one-shot Nash outcome is a separated network when the efficient outcome would have been a connected network. Neither will the dynamic version of the game always converge to the efficient outcome, if the efficient outcome were connected.

In the next part of the paper, we allow players to choose their identity, commitment to the chosen identity characteristic, as well as, their link strategies. First, we consider the case where the choice of identity itself does not add any utility. In this case, we still find that the Nash network of the one-shot game will be separated be identity or it will be a connected network with two identities or it will be a connected network with one identity. We find that the presence of multiple identities will vanish if we consider the efficient or the dynamic versions. Next, we allow the choice of identity itself to be of value depending on the set of other players who also choose the same identity. We find in this case that separation based on identity is indeed possible in both the one-shot game as well as in the dynamic version of the game. This segregation will be possible only if the benefits from identity are decreasing for some group sizes. In other words, players must have some intrinsic preference for some ideal identity-group size and this preference plays off against their desire to belong to the largest network possible. If the Nash equilibria of this game allow for multiple partitions of the network, then it must be that the benefits from identity has at least the same number of peaks as the number of possible Nash partitions. For a range of profit functions, the dynamic version of this game converges to the efficient Nash network.

Related Literature:

Previous work in economics including the choice identity includes work by Akerlof and Kranton (2000), Fryer and Jackson (2002), Currarini, Jackson, and Pin (2008), Sen (2006), Bisin and Verdier (2000), Esteban and Ray (1994). The Akerlof and Kranton (2000) model allows the self image (derived from identity) to affect the utility function, but they take as given which dimension of identity is salient, instead of allowing the individual to choose his salient identity. In Fryer and Jackson (2002), agents use identity to be able to sorting device and how this leads to biases. Currarini, Jackson, and Pin (2008), consider a matching model of friendship where agents have types/identities where players' utilities depend on the number of friends of the same type and those of different type. With this model they try to explain some empirical facts, among them, the presence of segregation. Since this paper focuses on how networks partition with the introduction of identity, it is also linked to the vast literature on club formation. Though most of this literature is not concerned with the how the networks evolve within a club, the paper by Page Jr. and Wooders (2007) bridges that gap. This work is strongly related to previous work Dev (2009) which also allows for players with multiple identities choosing commitments to identity as well as links. But unlike that paper we allow identity characteristics themselves to be chosen and we also allow link formation costs to be shared.

The network formation model used is the related to the literature on non-cooperative network formation models pioneered by Bala and Goyal (2000a) and Bala and Goyal (2000b), with related work on heterogenous players by Galeotti, Goyal, and Kamphorst (2006), Hojman and Szeidl (2008), Sarangi, Billand, and Bravard (2006), Galeotti (2006) and Gilles and Johnson (2000). Galeotti, Goyal, and Kamphorst (2006) allow society to be divided into groups and let connection within a group to be cheaper than connections across groups. Galeotti (2006) studies a model in which players are heterogeneous with respect to values and the costs of establishing a link. One type of heterogeneity considered is independent of partner or the cost/benefits depend only on the agent forming the link/receiving the benefit and the other heterogeneity is partner dependent.[3][4][5]

Within the network formation literature, this paper is also connected to the literature where links are

[3] The other strand in the network formation literature follows Jackson and Wolinsky (1996). The book by Jackson (2005) as well as Dutta and Jackson (2003) provide an excellent review of the literature.

[4] The recent paper by Page Jr. and Wooders (2009) unifies the two strands by suggesting a common framework with which to view all network games.

[5] Other important theoretical extensions of network formation models include Jackson and Dutta (2000), Watts (2001), Deroan (2003), Feri (2004), Kranton and Minehart (2001), Goyal and Joshi (2003), Goyal and Vega-Redondo (2005), Slikker and van den Nouweland (2001), Gilles and Johnson (2000), McBride (2006), Bramoulle and Kranton (2007).

formed based on transfers or consent, since these in both these strands of the literature two players are required to act for a link to be formed. The literature on transfers in networks includes important work by Bloch and Jackson (2007), Mutuswami and Winter (2002), and Currarini and Morelli (2000). The literature on consent in networks was developed from the paper by Gilles and Sarangi (2004).

The rest of the paper is organised as follows. Section 2 explains the model in detail and Section 3 outlines the assumptions used. In the next three sections, we present three versions of the game and their Nash equilibriums and dynamic equilibriums. Section 4 considers the game where identity characteristic is given exogenously and players choose commitments and links. Section 5 presents the game where identity is a variable of choice but this choice does not directly impact the utility of the player, whereas in Section 6, we allow the choice of identity to directly impact the utility. The next section presents the conclusion followed by the appendices with all the proofs.

2 Model

The set of all players is $N = \{1, ..., n\}$. Identity is defined along a single dimension which consists of a set of characteristics, $\{c_1, c_2\}$. Each persons i's identity, I_i consists of one of the characteristics. The identity profile of the population is given by I. I define a 'block' as a group made up of completely homogenous players who have the same characteristic. Each person has the following choices to make:

- Identity, player i chooses identity I_i such that, $I_i \in \{c_1, c_2\}$. In the paper, we study both cases, one where identity is fixed and another in which it is a variable of choice.

- Commitment, player i chooses commitment to his identity θ_i such that, $\theta_i \in [0, 1]$. In general, a higher commitment to any characteristic will make linking with people with the same characteristic cheaper but make links more expensive with people who don't have this characteristic. Let the $n \times 1$ matrix $\mathbf{\Theta}$ denote the commitment profile of the population.

- Link Offers, player i chooses how much he offers to pay for each link $l_i = \{l_{i1}, ..., l_{ii-1}, l_{ii} = 1, l_{ii+1}, ..., l_{in}\}$ where $l_{ij} \in [0, 1]$. A link between i and j is formed if $l_{ij} + l_{ji} \geq 1$. Let $\pounds = \{l_1, .., l_i, .., l_n\}$ be the population link profile. Let L_i be a n-dimensional vector such that

$$L_{ij} = \begin{cases} l_{ij}/(l_{ij} + l_{ji}) & \text{if } l_{ij} + l_{ji} \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Let the vector of dimnesion n, $g_i = \{g_{i,1}, .., g_{i,i-1}, g_{i,i}, .., g_{in}\}$, denote the direct links that i has, where $g_{i,k} = 1$ if $l_{ij} + l_{ji} \geq 0$ and 0 o.w. The links are undirected and if a link exists between l and k, they both have access to each other's information. The strategy for links generates a network denoted by g, where $g = \{g_1, ..., g_n\}$. Define $\overline{g} = cl(g)$ where an element of $\overline{g}$ is $\overline{g}_{kl} = \max\{g_{kl}, g_{lk}\}$ for all $l, k \in N$. We say a *path* exists between agents k and l if either $\overline{g}_{kl} = 1$ or there exist $j_1, ..., j_m$ such that $\overline{g}_{kj_1} = ... = \overline{g}_{lj_m} = 1$. A *path* is denoted by $k \overset{\overline{g}}{\longleftrightarrow} l$. A component within a graph g is $C(g) \subseteq N$ such that all agents within the component have a path connecting each other and there are no link going from any player in $C(g)$ to any player not in $C(g)$. A component is said to be *minimal* if deleting any link will lead to it not being a component anymore and a network is called *minimal* is all its components are minimal. A network is said to be *connected* if it has only one component made up of all players. A network is said to be *empty* if no player makes any links. A network is said to be *separated* if there are no links between players with different identities.

Let $N^i(g)$ be the set of players that i is linked to directly or indirectly in the network (assuming no decay). Besides the profit from the network, profit function of i will additionally depend on N^{si} which is the set containing all the players with the same identity as i. The profits are broken up into two components - the first is profits from the network and the other is profits from the choice of identity itself.

$$\Pi_i(\pounds, \Theta, I) = \pi(N^i(g), L_i, \Theta, I) + \phi(N^{si}) \tag{1}$$

As an illustration, network profits could be of the form:

$$\pi(N^i(g), L_i, \Theta, I) = f(N^i(g)) - \sum_{j:l_{ij}+l_{ji}\geq 1} \frac{l_{ij}}{l_{ij}+l_{ji}} c(\theta_i, \theta_j)$$

In this profit function, the first term denotes the benefits of belonging to the network and the second denotes the costs. The cost depends on commitments, but the share of cost paid depends on his offer of l_{ij}.

Definition 1 *The Nash equilibrium is a set of strategies $\{\pounds, \Theta, I\}$ which result in network g, such that for each player i*

$$\Pi(\pounds, \Theta, I) \geq \Pi(l'_i, \pounds_{-i}, \theta'_i, \Theta_{-i}, I'_i, I_{-i})$$

where $l'_i \neq l_i$ and/or $\theta'_i \neq \theta_i$ and/or $I'_i \neq I_i$

Definition 2 *The Efficient Nash Network is a network g supported by strategies $\{\pounds^*, \Theta^*, I^*\}$ such that:*

$$\{\pounds^*, \Theta^*, I^*\} \in argmax \sum_{i=1}^{n} \pi(N^i(g'), \pounds', \Theta', I)$$

where in the R.H.S, $\{\pounds, \Theta, I\}$ is a Nash equilibrium.

In the dynamic version of the game, one player is selected to act each period. In each period, first, the chosen player chooses his identity, commitment and link strategy. Link offers can be simple offers of how much he is willing to pay for the link; they can also be contingent on the aceptee changing to a certain commitment and/or identity. In the next part of the period, the other players simultaneously make the decision to accept or not his link strategy. The dynamic version will be said to have converged when no player has any incentive to change his identity/commitment or to change the component they belong to. Since, player form links by sharing costs, there will always be scope for renegotiating the share of the link bourne by each player; and in this way the dynamic game will never be free of movement within a component.

3 Assumptions for Network Profit Function

- **A1: Strictly increasing in $N^i(g)$.** Or that for any $x \subset N$, where $x \cap N^i(g) = \phi$, and for any L_i, Θ and I,

$$\pi(N^i(g) \cup x, L_i, \Theta, I) > \pi(N^i(g), L_i, \Theta, I)$$

- **A2: Strictly decreasing in all elements L_i.** Moreover, the marginal effect of the L_{ij} depends on $\{\theta_i, I_i\}$ and $\{\theta_j, I_j\}$. If identity is the same, the higher are the commitments, the lower is the effect. If identity is different, the higher are the commitments, the higher is the effect.

- **A3: Only Adding a Profitable Link is Profitable.** Suppose player i currently makes profits of $\pi(N^i(g), L_i, \Theta, I) \geq 0$, and $\exists$ a neighbourhood X, a strategy Y, such that

$$X \cap N^i(g) = \emptyset$$
$$Y_j > 0 \Rightarrow L_{ij} = 0$$

  then

$$\pi(N^i(g) \cup X, L_i + Y, \Theta, I) > \pi(N^i(g), L_i, \Theta, I) \text{ iff } \pi(X, Y, \Theta, I) > 0$$

  Where $L_i + Y$ is a simple element by element addition and we need to keep in mind that might not yield the set of neighbours $N^i(g) \cup X$ in the new network.

- **A4: Player Labels are Irrelevant**

  if x^p denotes a permutation of vector x, then

$$\pi(N^i(g), L_i, \Theta, I) = \pi(N^i(g), L_{ii}, L_i^p, \theta, \Theta_{-i}^p, I_i, I_{-i}^p)$$

The first assumption is about the benefits of being in the network and it implies that each link is valuable to every player. In other words any group of players has something of value to offer to each other player and also that each player in the neighbourhood of say player y adds value to y irrespective of who the other players in y's neighbourhood are. The second assumption is about the costs and it implies that costs are increasing in the 'amounts' offered. The impact of the link strategy on costs also depends on the identity as well as commitment choices. For any link offer between players of the same identity; the impact on costs will be lower the higher are (any or both of) the commitments. On the other hand, for any link offer between players of different identity, the impact on costs will be higher; the higher are (any or both of) the commitments.

The next assumption concerns both cost and benefits. It first implies that adding a link which is profitable is profitable for any player who is not already linked to the players accessed by this link. In other words, combining two individually profitable link strategies, will yield higher profits than either of the two strategies individually. It also implies that if a player has a link strategy which includes offers of links to more than one player, each one of these link decisions made by the player is individually profitable irrespective of the rest of the player's strategy. The benefits from each link decision within a player's linking strategy might well depend on the entire strategy, but even so, each one of these link decisions must at least yield positive profits in isolation. With this assumption we want to limit the extent of negative externalities.

The fourth assumption states that for any strategy, the benefits only depend on the neighbourhood accessed. The costs only depend on the offers made, the choice of commitment by the player and the choice of commitments by the players to whom the link offers are made. The name labels of the players are irrelevant.

The last assumption, A5, is used to facilitate the discussion in the last sections where we allow players to choose identity and for this choice to directly impact the utility. It says that all players are ex-ante equal. In other words, all players are the same as far as their informational value to the network is concerned or that profit from the network depends only on the number of people linked to. It helps to keep the focus on the choice of identity rather than on player values.

A5: Equal Values If each player has the same value to add to any network we get:

$$\pi(N^i(g), L_i, \Theta, I) = \pi(\#N^i(g), L_i, \Theta, I)$$

4 Fluid Commitments

In this section we study the game where only the choice of commitment is possible or in other words, commitment is fluid but the population identity profile, I, is given exogenously. The choices available to each player i then are, commitment strategy, θ_i, and link strategy, l_i. We first look at the one-shot static Nash Network and its properties.

4.1 The Static Game

The following proposition shows that the Nash Network will have one of three types of connectivity - connected; separated with each block forming either a component or all players of the block remaining singletons; and finally, empty with no links being formed.[6]

Proposition 1 *Under assumptions A1-A4, the Nash Network of the game, where players choose their commitment levels and link strategies, will have one of the following structures:*

- *Connected*

[6] If we were to allow for more than two characteristics, another possible equilibrium structure would involve some blocks being connected, and all other blocks isolated from each other and the connected blocks.

- *Separated, where each block will either form a component or each player of the block remains a singleton*

- *Empty*

The proof is presented in the form of the following lemma. The lemma says that each block either belongs to the same component or all members of the block are singletons. The intuition for this lemma is as following. Suppose $i \in B$, where B is a block. Further suppose that i is linked with k. Now we know from our assumptions that it must be profitable for i to link to k and similarly, it must be profitable for k to link to i. Individually, both of these strategies are not feasible. But, the sum of these two strategies is feasible, because it involves sponsoring the entire link between i and k. Moreover, the sum of the strategies must also be profitable by A3. If there is some player $j \in B$ who is not linked to i, he could use the sum of the strategies explained above to profitably link to i. Given this lemma, it is clear that either the two blocks are separated or linked.

Lemma 1 *In a Nash Network, each block belongs either to the same component or everyone from the block is a singleton*

Proof. Suppose there are players i and j, and a block B and components C and C' such that $\{i, j\} \in B$ but $i \in C$ and $j \in C'$.

- Suppose C' is a singleton. But C is not, and i participates in a link with $k \in B$ in C. Let π_{ik} denote i's profits from linking to k, then by A3

  $$\pi_{ik} = \pi(N^{ik}(g), L_i^k, \theta_i, \theta_k, \Theta_{\{-i,-k\}}) \geq 0$$

  where $N^{ik}(g)$ are the people accessed by i through this link and the strategy L_i^k is such that $L_{ik}^k = L_{ik}$ and all other elements are zero.

  If j were to choose a link strategy L_j' which had only one non-zero element, $L_{ji}' = L_{ik}$, and choose $\theta_j = \theta_k$, and if by doing this he could observe $N^{ik}(g)$, then by A4 we would have:

  $$\pi_{ji}' = \pi(N^{ik}(g), L_j', \theta_i, \theta_j = \theta_k, \Theta_{\{-i,-j\}}) = \pi_{ik}$$

  Similarly, let π_{ki} denote k's profits from linking to i, then by A3

  $$\pi_{ki} = \pi(N^{ki}(g), L_k^i, \theta_i, \theta_k, \Theta_{\{-i,-k\}}) \geq 0$$

  Now again, if j were to choose a link strategy L_j'' which had only one non-zero element, $L_{ji}' = L_{ki}$, and choose $\theta_j = \theta_k$, and if by doing this he could observe $N^{ki}(g)$, then by A4 we would have:

  $$\pi_{ji}'' = \pi(N^{ki}(g), L_j'', \theta_i, \theta_j = \theta_k, \Theta_{\{-i,-j\}}) = \pi_{ki}$$

Let π_{ji} denote the profit to j from linking to i by choosing L_j''' where he chooses to make no other links except $l_{ji}''' = 1$ and $\theta_j' = 1$

$$\begin{aligned}\pi_{ji} = \pi(C, L_j''', \theta_j', \Theta_{-j}) &\geq \pi(N^{ik}(g) + N^{ki}(g), L_j' + L_j'', \theta_j'', \Theta_{-j}) \\ &\geq \max\{\pi_{ik}, \pi_{ki}\} \geq 0\end{aligned}$$

{where $\theta_j'' = \theta_k$} The second inequality uses assumptions 1, 2 and 3.

A similar conclusion would hold if it were the case that $k \notin B$.

- Now suppose C' is not a singleton. If j only links to players from B, then j must have $\theta_j = 1$ and by the previous logic, he could add a profitable link to i. Similarly, if i only links with players from B, i could profitably add a link to j. If both i and j are linking to players outside B, then changing θ's is costly. Suppose j links to j' and i links with i', where j' and i' do not belong to B. Suppose, wlog, $\theta_j = min\{\theta_j, \theta_j', \theta_i, \theta_i'\}$. Then j could add a link to i' by offering to pay for all of it and this additional link by j would add to his profits.

■

Next, within the set of Nash Networks, we wish to find the ones which are efficient. Under assumptions A1-A4, the Efficient Nash Network of the game, where players choose their commitment levels and link strategies, will be such that:

- A block is internally connected iff there exists any Nash network in which the block is connected.(R1)
- The existence of a Connected Nash equilibrium will not imply the connected network is efficient. Though if it is that the efficient network is connected, there will be exactly one player from each block participating in an external link.

To see this, firstly observe that, if in any Nash network a block is internally connected, each individual of that block should prefer to be linked over not having any links at all. In other words, the connected block will be increase total welfare over an empty network. Next, if the efficient network is connected, we must have only a single player participating in the external link to minimise the overall costs. (Note that in the following text we will be referring to the first condition, that a block is internally connected iff there exists any Nash network in which the block is connected, as R1)

4.2 The Dynamic Results

We now consider the dynamic version of the game, where one player is selected to act each period. This player chooses his commitment and link strategy. Link offers can be simple offers of how much he is willing to pay for the link; offers can also be contingent on the acceptee changing to a certain commitment. The acceptees simultaneously choose whether to accept or reject the offers.

The next proposition shows how the dynamic game will evolve. Firstly, the dynamic version of the game will converge to the empty network if that is the only static Nash equilibrium. This is the case where costs are just too high to sustain any links at all. Next, for any block, if there exists at least one static Nash network where the block is connected, this block will converge to be connected in the dynamic version of the game as well. We also show that if the dynamic process converges to a connected network, each block will be internally linked. Finally, if the static Nash network could be both separated or connected, the dynamic version will transition to connected iff there exists a connected network such that just breaking the inter-block links (between the two highest valuation blocks) and giving everyone a theta of 1 and keeping everything else the same, leaves at least one of the blocks in equilibrium.

Proposition 2 *Under the assumptions, A1-A4, the dynamic version of the fluid commitment game will converge to:*

- *The empty network, if that is the only static Nash equilibrium.*
- *A block will be connected if R1 holds.*
- *If R1 holds for at least one block and there is no Connected Nash equilibrium, the dynamic game will converge to a separated network.*
- *If R1 holds for each block, the network will transition to a connected network only if there exists a connected network such that just breaking the inter-block links and giving everyone a theta of 1 and keeping everything else the same, leaves at least one of the blocks in equilibrium.*

The proof of this proposition is presented in Appendix A. It is presented in the form of a series of lemmas. The first lemma shows that if a block is connected in any Nash equilibrium, it must be possible to construct a Nash equilibrium where a given player has just one link. To give an intuition for the proof, suppose not, i.e., there existed a player who was willing to pay the maximum he could to link to the rest of the players in his block, but no other player could profitably participate in this link with him. In other words, using A3, this entire link between the player and his block would be unprofitable. Which would mean that if we were to invent a new player in this block, he would find it unprofitable to sponsor an entire link to the existing block. But we know that this block can be profitably linked and any new player added to the block will find it profitable to sponsor an entire link to the block. Hence, it must be that there exists a profitable and feasible strategy where a player makes a single link to the rest of the block. Knowing that there is some link offer at which each player of this block can be individually linked to, in the second lemma, we show that the first player from this block called on to act will have a link strategy offer in the form of a star network with him linking to all players of the block. Which means that in any dynamic setting this block could always converge to being connected - the player chosen to act will just propose to form links with everyone. The proof is illustrated in the Figures 2 and 3, where all players have the same identity and characteristics, and

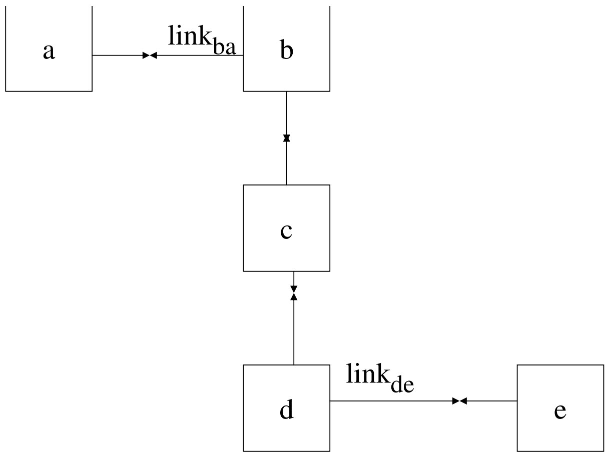

Figure 2: Initial Connected Network

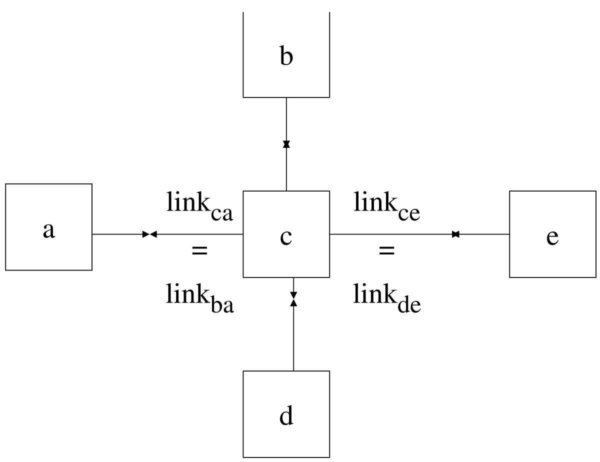

Figure 3: Possible Star Network

they are initially connected with player 'a' linking to player 'b' and player 'e' linking to player 'd'. This initial network could transform to that in Figure 3 is players 'a' and 'e', switch to linking with player 'c'.

If the dynamic network is connected, it will converge to a network where there each block is internally connected and there is one player from each block participating in the external link. To get an intuition of this, consider two players from the same block participating in external links. If one of them were to link to the other, the player's benefit in terms of number of players linked to would not change, but it might be that this internal link is more expensive than his current external link. But then given the dynamic nature of the game, there would come a time when one of these players will be making zero profits from the external link. And if they are chosen to move at a time when they are making zero profits from the external link, they would always choose to drop the external and form the cheaper internal link.

Finally we show that if the dynamic network is currently separated, but a Connected Nash equilibrium exists, then the transition from separated to connected will take place only if breaking the inter-block links in one of the connected equilibriums and giving everyone in one of the blocks a theta of 1 is also a static Nash equilibrium. This is so because the player who will offer to make the external link will have the opportunity to adjust his internal link as well. The player making the offer of the external link will reduce his commitment to make this external link more profitable, at the same time, with this change in commitment he might want to pay less for his internal link. But the player who accepts the offer of the external link will not be able to make this adjustment to his internal link. The player who is offered the external link, will then accept this offer only if such an acceptance increases his overall profits even though his profit from the internal link has reduced.

5 Fluid Identity, Choice of Identity has no Direct Impact on Utility

From this section on we allow players to chose their identity as well as commitments and links. They don't have any characteristics given, but rather they choose which identity they want to adapt to and their level of commitment to the identity chosen. As explained in the model, this choice of identity is to choose either of the two characteristics in the set of possible characteristics; $I_i \in \{c_1, c_2\}$.

In this section, we study the special case where the choice of identity itself has no implications on the utility or that $\phi(N^{si}) = 0$ for all players i and all N^{si}. The assumptions on network profits remain the same.

5.1 The Static Game

We show that the static Nash network will be one of three types. It will either be empty, with no links being formed and identity being indeterminate.[7] The Nash network could be Separated, but under very special

[7]This equilibrium would always be possible when the cost of a single link to a person of the same identity choosing any commitment below $\underline{\theta}$ was too expensive to leave any positive profits. Similary, when the cost of a single link to a person of a different identity choosing any commitment above $\overline{\theta}$ was too expensive to leave any positive profits. It will involve all players being indifferent in the choice of identity but choosing some $\underline{\theta} \leq \theta \leq \overline{\theta}$.

conditions, such that the profit function and the network structure must be so that everyone makes exactly zero profits. Lastly, we could see a Connected Nash network with all the players choosing the same identity and also a commitment of one or a Connected Nash network with players choosing either identity.

Proposition 3 *Under the assumptions A1-A4, when players choose identity, commitment and links, but* $\phi(.) = 0$*, the static Nash network will have one of the following structures:*

- *Empty network.*
- *Separated, with some of the players choosing one identity and the rest of the players choose the other identity. Moreover the network structure and profit function must be such that all players make zero profits.*
- *Connected with two identity blocks.*
- *Connected with all players choosing the same identity.*

The proof is presented in the Appendix B in a series of lemmas. The first two lemmas together show that the profits for each player must be zero in a Separated Nash network. In particular, the first lemma shows that in a Separated Nash network, all players making a single link, must make the same profits. To give an intuition for the proof, consider player i participating in a single link in the first block making less profit than a player i' in the second block participating in a single link. From assumption A3, we know then that sponsoring an entire link to the second block will be more profitable than the profits of i'. Thus, i could then increase his profits by switching his identity and offering an entire link to the other block. In other words, all players in either block making a single link, must be making the same profits. The second lemma just extends the same idea to conclude that all players must be making exactly the same profits - else, a player making lower profits in one block would switch to changing his identity and offering a link to the other block where higher profits are to be made. This leads us into the third lemma which says that any deviation should also give these same profits. Finally, the next lemma shows that in a Separated Nash network, the blocks must be such that all players make exactly zero profits. We already know that everyone makes the same profits, this lemma shows these profits must be zero. Suppose not, then a player making a single link will be making positive profits, moreover, the total profits from this (and any other link) must equal the profits made by any player. Or using assumption A3, we get that player participating in the link with a single other player must make zero profits from that link. In other words, he must make profits from other links. But since each sequence of link ends and begins with a player making a single link, it is not possible to sustain positive profits. In other words, a Separated Nash network will be possible only if the profit function is such that it allows for a split of the population into two identity blocks where each player would make exactly zero profits.

The final lemma in this section outlines the conditions for the presence of a Connected Nash network with two identities. For a partition to be supported as Connected Nash Network, we need at least one player each from both blocks that is participating in an external as well as internal link. And we must have both these contributions strictly positive. These players must not want to severe either connection or to switch identity. Further more, the players linking to these players making external links should better off keeping those links than severing them.

To concretize the concept of the Separated network , consider the following figure. In the Figure 4,

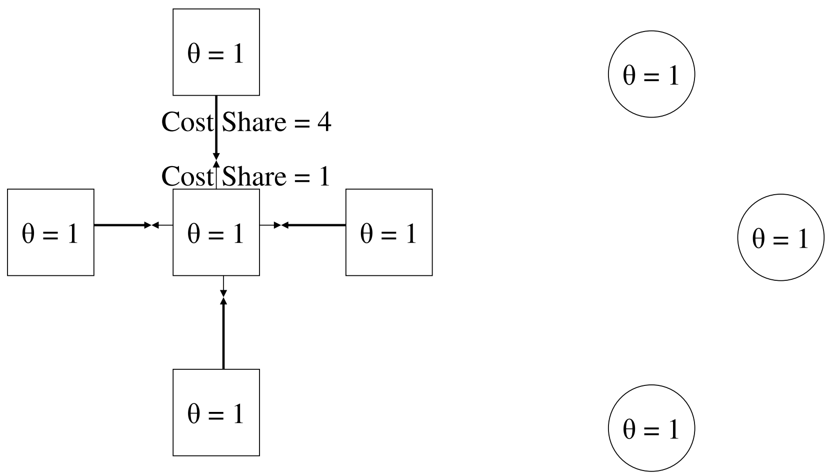

Figure 4: Separated Network

players can choose to be either a Square or a Circle. Their profits are from the number of players observed less the sum of link costs. In particular, for this separated network to be sustained as Nash, it must be that link costs between players of similar identity are exactly 5. And we must have the center of the Square star paying 1 for each of his 4 links, whereas, the periphery players pay 4. In other words, each Square player is linked to 4 other players and pays 4 for his links, making zero profits. The Circle players make zero profits since they are unlinked, but deviating to being a Square and linking to the Square star would still leave them with zero profits.

We next consider efficient Nash networks. Where we find that the Efficient Nash Network will be either

- Connected with a single identity block or
- Empty with players choosing either identity.

Since choice of two identities only serves to distort the costs of link formation, the Efficient Nash Network must either be connected with a single identity block or it could be empty, but in that case the choice of identity is irrelevant to network payoffs.

5.2 The Dynamic Results

We now consider the dynamic version of the game. Again one player is selected to initiate each period. This player chooses his identity, commitment and links strategy. Link offers can simple offers of how much he is willing to pay for the link, offers can also be contingent on the aceptee changing to a certain commitment and/or identity. The other players choose to accept or not his link strategy.

The important result we get out of this setup is that when choice of identity has no direct impact on utility, no equilibrium with links being formed will feature two identities. In all cases, the dynamic game will converge to everyone choosing the same identity and the network being connected. The only other possibility is the empty network with players indifferent in the choice of identity - but this will be possible only if the static Nash equilibrium allows for only the empty network.

Proposition 4 *Assuming A1 - A4, when players choose identity, commitment and links, but* $\phi(.) = 0$*, in the dynamic version of the game the equilibriums with two identities will not survive. The only exception is the case where the only static Nash network was the empty network.*

The proof is presented in a series of lemmas. The first shows the impossibility of separated identity blocks. This follows easily, once we see that the player participating in a single link in one of the blocks, can change his identity and follow the same link strategy with the other block. This will lead to one of the identity blocks unraveling. The next three lemmas show that connected identity blocks will also not survive the dynamic process. The first of these three shows that a player participating in an external link will have exactly one internal link. If a player with a $\theta < 1$ participates in two internal links, one of the recipients of the internal link will find it profitable to link to the other recipient, hence, linking to a player with $\theta = 1$ and lowering his costs. The next lemma shows that there will be only one external link between the two identity blocks. If there were more than one external link, one of the players participating in the external link would prefer to break the external link and link to the other player with the same identity participating in the other external link. Finally the last lemma shows that connected identity blocks will not survive the dynamic process. This will be so because one of the players participating in the external link will have the incentive to disinvest from the internal link and switch to the other identity, which will set off a chain reaction of everyone from the identity block switching their identities.

In other words, in this scenario where players choose identity, but this choice has no direct impact on the utility function, we will see the dynamic game converge to the Efficient Nash network.

6 Fluid Identity, Choice of Identity has Direct Impact on Utility

Till now we have that the choice of identity could lead to two separate identities in the static Nash equilibrium, and these separate identities would vanish only under the dynamic version of the game. We now drop the assumption that the choice of identity has no direct impact on utility. It is but obvious that this addition of direct benefits from identity will allow more Nash equilibria where the network is separated by identity, the interesting question is relating different kinds of direct benefits from identity to different possible sets of Nash equilibria.

The benefits of identity will mean that the profit function of i will now additionally depend on N^{si} which is the set containing all the players with the same identity as i. The profits are broken up into two components - the first is profits from the network and the other is profits from the choice of identity itself. For the rest of this section, to focus attention on the choice of identity rather than on individual values, we will work with the assumption A5 or that profit only depends on the number of people. Or using small caps to denote the number of elements in the set, we get that profits must be:

$$\Pi_i(g,\Theta,I) = \pi(n^i(g), L_i, \Theta, I) + \phi(n^{si}) \tag{2}$$

6.1 The Static Game

Let's try to find conditions where the Nash network will feature two identities under the assumption that identity does affect utility. Suppose in a Nash equilibrium there are two identity blocks, block 1 consists of the set B_1 (with b_1 players) and block 2 consists of the set B_2 (with b_2 players); such that, $b_1 \leq b_2$. Any player trying to switch from one identity block to another will have to sponsor a whole link to someone from the new identity block and will have no other links, let's denote this strategy by L^s. All players considering such a switch to block B_i will make the same profits, they will sponsor one whole link and observe $b_i + 1$ players in all; let us denote the profits of switching to block B_i by $\pi^s(b_i + 1)$. If any $i \in B_1$ decides to switch to block 2, his profit from this deviation must be less than his current profit or:

$$\pi^s(b_2 + 1) + \phi(b_2 + 1) \leq \pi(b_1, L_i, \Theta, I) + \phi(b_1)$$

(where the input Θ is a vector of one's)

Similarly for any player $j \in B_2$ we have

$$\pi^s(b_1 + 1) + \phi(b_1 + 1) \leq \pi(b_2, L_j, \Theta, I) + \phi(b_2)$$

Both these above equations must be satisfied for the smallest profit maker, $i \in B_1$ and the smallest profit maker $j \in B_2$. In particular, for any network size, let's choose the network configuration which gives the maximum such smallest profit and let's call these profits, for block size b_i as $\pi^e(b_i)$. Then as long as the above two conditions are satisfied for $\pi^e(b_1)$ and $\pi^e(b_2)$, then we know there is at least one network structure that will allow the partition to exist.

Now let's define a new function for all $b_i \leq n/2$

$$\psi(b_i) = \phi(b_i) - \phi(n - b_i + 1)$$

Also define these bounds

$$\overline{\psi(b_i)} = \pi^s(n - b_i + 1) - \pi^e(b_i)$$

and

$$\underline{\psi(b_i)} = \pi^e(n - b_i) - \pi^s(b_i + 1)$$

Notice $\overline{\psi(b_i)} \geq \underline{\psi(b_i)}$ for all b_i. Also that $\overline{\psi(b_i)}$ and $\underline{\psi(b_i)}$ are both decreasing. But, at $n/2$ they are both exactly the same absolute value. The first lemma in Appendix D shows that $\overline{\psi(b_i)} \geq \underline{\psi(b_i)}$ for all $b_i \leq n/2$.

The next proposition shows the kinds of $\phi(.)$ functions under which a separated identity blocks are possible. It is interesting to note that if network profits were always zero and the only profits were from identity directly; b_1, b_2 could be supported as a Separated Nash equilibrium, if and only if $\phi(b_1) \geq \phi(b_2+1)$ and $\phi(b_2) \geq \phi(b_1+1)$. In particular, if ϕ were single-peaked, then the only Separated Nash equilibria possible would be at $n/2$. With that in mind, we see that in the general case when networks are beneficial, we see that Separated Nash equilibria will not necessarily be at $n/2$, if fact, we see that when ϕ is symmetric around $n/2$ no Separated equilibria is possible. Further we see the possibility of multiple Separated equilibria if ϕ has more than one peak.

Proposition 5 *Under A1-A5 and profits are as defined in equation 2; for any partition $b_i, n-b_i$ to be supported as a Separated Nash equilibrium, we must have $\psi(b_i) \geq \overline{\psi(b_i)}$ and $\psi(b_i+1) \leq \underline{\psi(b_i+1)}$. For $b_i = n/2$ to be a partition supported by a Nash equilibrium, we must have $\psi(n/2) \geq \overline{\psi(n/2)}$. For the entire network to be connected with a single identity we need, $\psi(0) \leq \underline{\psi(0)}$.*

- *If ϕ is symmetric around $n/2$, no Separated Nash equilibrium will be possible.*
- *If ϕ is concave, at most one partition could be supported as Separated Nash equilibrium. Further, for a partition to exist, the peak of ϕ must be before $n/2$.*
- *If ϕ is convex, a Separated Nash equilibrium is possible only if the lowest point is beyond $n/2$*
- *If ϕ is such that it can be partitioned into regions that are concave around peaks and convex around troughs, the number of Separated Nash equilibrium will be at most the number of peaks in ϕ.*

The proof is presented in the Appendix D as a series of lemmas. We first show that $\overline{\psi(b_i)} \geq \underline{\psi(b_i)}$. We prove this by showing that $\pi^s(n-b_i+1) \geq \pi^e(n-b_i)$ and $\pi^e(b_i) \leq \pi^s(b_i+1)$. We show the first inequality by showing that the player making the least profits makes a profitable link with another player, where the sum of these two strategies would be where a player outside the block sponsored an entire link to the $n-b_i$ players and made profits greater than the lowest profit maker in $n-b_i$ block. Or $\pi^s(n-b_i+1)$ is greater than $\pi^e(n-b_i)$. The other inequality is proven similarly.

Next we claim that for a Separated Nash equilibrium to exist, $\phi(.)$ cannot always be increasing. We already know that sponsoring an entire link to a larger block yields higher network profits to the smallest network profit maker in the smaller block, if the larger block also yielded higher identity benefits, the smaller block would just unravel.

Now if $\phi(.)$ were symmetric around $n/2$, then we would find that $\psi(.)$ would always be zero; which rules out the possibility of Separated Nash equilibria. We next consider the case where $\phi(.)$ is concave. Here, we show that only if the peak occurs before $n/2$ will $\psi(.)$ be positive in the relevant region, and otherwise it will be negative in the relevant region. Hence, a Separated Nash equilibrium might exist only if $\phi(.)$ peaks before $n/2$. Moreover, there could be only one such partition where a Separated Nash equilibrium is possible. Similarly, if $\phi(.)$ is convex, a necessary condition for the Separated Nash equilibrium to exist will be that the trough of $\phi(.)$ be after $n/2$. Finally, we show that the number of separated equilibria will be less than the peaks of $\phi(.)$, since those correspond to the maximum number of changes in inflexion of the $\psi(.)$ function.

Next we will check for the partitions at which Connected Nash equilibria could exist. Assuming a Connected Nash equilibria exists for the partition under fixed identity, for fluid identity we have three possible deviations from each block where the player switched identity. The three deviations are by the player who makes the external link, the player who makes the internal link with the player making the external link and finally a player from the rest of the block. Let me call these players - i, i' and i''. (the counterparts in the other block are j, j' and j''). Suppose when i switches his identity and deviates to strategy L_i', θ_i', he makes $\delta_i(b_i, \pounds, L_i', \theta_i')$ more in network profits from the new network as compared to the old one. For any given network, let his best increase in profits be captured by $\delta_i(b_i, \pounds)$. Remember, this player can switch identity and maintain his link with j from the other block or he could sponsor an entire new link with some player from the other block who has a $\theta = 1$, which one is a better strategy would depend on the exact profit function. Further, i could choose to keep his link with i', again this would depend on how sensitive network profits are choice of θ's. For player i', we similarly define $\delta_{i'}(b_i, \pounds)$. Finally, we have $\delta_{i''}(b_i, \pounds)$, which would be highest possible benefit from switching identity for a player making internal links only with other players who have $\theta = 1$. If each such player in the original network was in effect paying for less than one link, then these benefits of switching would always be negative. For a partition at $b_i, n - b_i$ with the network structure of $\pounds$, to be a supported by a Connected Nash network, we would need $\psi(b_i)$ to be greater than:

$$\overline{\psi(b_i, \pounds)} = \max\{\delta_i(b_i, \pounds), \delta_{i'}(b_i, \pounds), \delta_{i''}(b_i, \pounds)\}$$

Defining similarly the bounds for other block, we would also need $\psi(b_i + 1)$ to be less than:

$$\underline{\psi(b_i + 1, \pounds)} = \min\{-\delta_j(b_i, \pounds), -\delta_{j'}(b_i, \pounds), -\delta_{j''}(b_i, \pounds)\}$$

Proposition 6 *A Connected Nash network at $b_i, n - b_i$ will emerge as one of the Nash equilibria if there exists some link strategy $\pounds$ and the corresponding optimal choice of Θ such that*

- *$\pounds$, Θ comprise a Connected Nash equilibrium in the game with fixed identity*
- *$\psi(b_i) \geq \overline{\psi(b_i, \pounds)}$ and $\psi(b_i + 1) \leq \underline{\psi(b_i + 1, \pounds)}$*

Proof. For a Connected Nash network to exist at a particular partition, we must have that that network and commitment strategy is an equilibrium for the game with fixed identity. Further, we need to ensure that no player makes any profits from switching identity. ■

Corollary 1 *If at $n/2$ there exists a Connected Nash equilibrium of the game with fixed identity and if ϕ is decreasing at $n/2$, then there will be Connected Nash equilibrium at $n/2$ for the game with fluid identity. The second condition is met for instance in ϕ symmetric, in ϕ concave with the maximum before $n/2$ and in ϕ convex with the minimum after $n/2$.*

As opposed to separated networks, we cannot draw a connection between the number of humps in the ϕ function to the number of possible Connected Nash equilibria. In fact, the flatter is the ϕ function, the more are the possible connected equilibria. The extreme case being of ϕ being absolutely flat and then all the Connected Nash networks of the game with fixed identity will be carried over as connected equilibria in the game with fluid identity.

We next consider the Efficient Nash Networks in this setup. Assuming that a non-empty Nash network exists, Efficient Nash Networks will be such that for:

- If ϕ is symmetric around $n/2$, the efficient network will be a connected network with block sizes of $n/2, n/2$.
- If ϕ is concave, with the peak occurring before $n/2$, the connected network at $n/2$ will be efficient for large n, whereas, for small n it will be the Separated Nash network if it exists.
- If ϕ is first convex, with the peak occurring after $n/2$, the connected network at n with a single identity will be efficient.

For ϕ with multiple peaks, finding efficient Nash networks is more tedious as we need to rank amongst the many possible separated networks and the connected networks.

6.2 The Dynamic Results

The dynamic game is the same as before where again the selected player chooses identity, commitment and offers. The recipients of the offer then chooses his identity, commitment and whether to accept the offer or not. We continue with assumption A5 as well as that the benefits from identity depend only on the number of players with the same identity.

We are now interested in finding out which Nash equilibria will survive as Dynamic equilibria. Also are multiple Dynamic equilirbia possible - i.e. starting at two different initial networks, will the final network be different? Clearly, under our current assumptions, some Nash equilibria partition will be ruled out under the dynamic game. One, because, now switching identity and component does not require sponsoring the entire link and; two, given that under dynamic game, block deviations are possible.

Let L^{min} denote the minimum that a player with a single link has to pay for his link with a player of same identity and commitment of 1. If the chosen player is with a single link, he will either make an offer of L^{min} to his current link; or he will choose to switch his identity and make a link with some player of the other block, again offering L^{min}. Given this and letting I^i denote the identity profile where player i switched identity; we know that necessary bounds for the Separated Dynamic equilibrium are

$$\overline{\psi^{DE}(b_i)} = \pi(n - b_i + 1, L^{min}, \Theta, I^i) - \pi(b_i, L^{min}, \Theta, I)$$

and

$$\underline{\psi^{DE}(b_i)} = \pi(n - b_i, L^{min}, \Theta, I) - \pi(b_i + 1, L^{min}, \Theta, I^i)$$

In other words, in any Separated Dynamic equilibrium, it must be that $\psi(b_i) \geq \overline{\psi^{DE}(b_i)}$ and $\psi(b_i + 1) \leq \underline{\psi^{DE}(b_i + 1)}$

Proposition 7 *Assuming A1-A5 as well as that for any B_1, B_2 and strategy profiles L_1, L_2:*

$$\pi(B_1, L_1, \Theta, I) - \pi(B_1, L_2, \Theta, I) = \pi(B_2, L_1, \Theta, I) - \pi(B_2, L_2, \Theta, I)$$

Then we have that the equilibrium of the dynamic game if it exists is unique and is a Nash equilibria of the static game. Further, given that the dynamic network converges;

- *If ϕ is symmetric around $n/2$, the dynamic equilibrium will be a connected network with block sizes of $n/2, n/2$.*
- *If ϕ is concave, with the peak occurring before $n/2$, the dynamic equilibrium will be connected at $n/2$ will be efficient for large n, whereas, for small n it will be the Separated Nash network.*
- *If ϕ is convex, with the peak occurring after $n/2$, the connected network at n with a single identity will be the only possible dynamic equilibrium.*

The condition used in the proposition, is essentially saying that change in network profits from changing link strategy should be the same irrespective of players linked to. One way to attain this restriction is to consider the set of profit functions that can be separated into benefits and costs.

The proof is presented in Appendix E. We start by showing that the dynamic equilibrium must be a Nash equilibrium. Next we show that the dynamic equilibrium allows for groups of players to deviate to linking to the other identity block. This is possible in a case where a player is the center of star with say k spokes. Now when this player moves in the dynamic game, he can ask all these k players to change identity and keep their link; and he can himself change identity and link to the other identity block. In this manner $k+1$ players could move to the other block. If there were no star with k spokes right now, we show that at some point of time such a star must exist. Consider a player making a single link, we know that given the nature of the dynamic game, this player must be making zero (or the minimum possible) profits at some time. When at such a time this player moves, he is indifferent to which player he links to - which gives rise to the possibility of the star. Finally, we show that the dynamic equilibrium must be unique. If it were connected, we know players (or groups of players) will keep changing their identities till no improvements can be made from identity benefits. Similarly, if the dynamic game converged to being separated, and there were another separated dynamic equilibrium which gave higher profits, the original separation would not survive the dynamic process.

The proposition also points out the link of the dynamic game with that of the efficient Nash equilibria for different types of identity benefits. We see that if ϕ is symmetric around $n/2$, the dynamic network will be a connected network with block sizes of $n/2, n/2$. Since we know that no Separated Nash equilibrium exists for this case, the dynamic equilibrium must be connected; and the only network at which identity benefits cannot be improved upon is where block sizes are $n/2, n/2$. If ϕ is concave, with the peak occurring before $n/2$, we find that the dynamic network as well as the efficient Nash network, will be connected at $n/2$ will be efficient for large n, whereas, for small n it will be the Separated Nash network if such a network exists. It is important to note that for the dynamic network to be connected, a larger set of players might be required than for the efficient Nash network to be connected. If ϕ is convex, with the peak occurring after $n/2$, the connected network at n with a single identity will be the only possible dynamic network. This must be the case because at any separation, the smaller identity block will be better of switching identity and joining a larger network and getting larger identity benefits. Finally, for ϕ with multiple peaks, we know there are multiple Nash equilibria possible. Here, the dynamic equilibria if it exists will be the one where individual profits are the maximum. Hence, we see that the dynamic equilibrium often turns out to be the efficient Nash equilibrium.

7 Conclusion

If the *only* desire is to have the maximum number of connections, the choice of identity cannot divide the population in the long run, (though it may very likely lead to two connected identity groups in the Nash equilibrium). Only if the desire to have the maximum number of connections is coupled with the desire to have the optimal number of same identity-group members would we see a division of the network both in the Nash equilibrium and in the dynamic setting.

To sustain a partition, the benefits from identity must be such that the smaller identity-group must have very strong benefits from identity, so much so that switching to the larger identity-group (and having more connections in the case of a separated equilibrium) does not attract them. On the other hand, the larger-group should prefer to stay in the large group for the benefits of larger connections and no one from this group should want to deviate to the smaller identity-group.

If we try to extrapolate from the model to the actual world, it would follow that the long run existence of distinct identity groups stems from an innate desire within each person for such groups; even though the absence of this desire and the lack of different identity-groups would increase benefits for all involved. Another way of looking at it is, that it is not that these separate identities were simply imposed on us; it is rather that as individuals and as a collective, we have chosen to have and retain these separate identities.

A Proof for Fluid Commitments, The Dynamic Game

The proof of the proposition on the presented in the following series of lemmas.

Lemma 2 *If R1 holds for a block, for any player $i \in B$, there must be a network which satisfies R1 such that i forms only one link.*

Proof. Suppose not, then $\exists i$, such that in no network configuration is it possible for i to just have one link. In other words for player $j \neq i$ and any strategy L_i^* where $L_{ij}^* > 0$ and $L_{ik}^* = 0$ for $k \neq j$, such that

$$\pi(N/i, L_i^*, \mathbf{1}) = 0$$

but

$$\pi(i, L_j^*, \mathbf{1}) < 0$$

where L_j^* is such that $L_{ji}^* = 1 - L_{ij}^*$ and $L_{jk}^* = 0$ for $k \neq i$. i.e. even though i is willing to pay the maximum possible to link with the rest of the players using just one link to player j, player j can not profitably link to him. And this is true for any player $j \neq i$

But since R1 is satisfied and we assume A3, there must exist some Nash network where the block is connected or i forms some profitable links. Suppose in one such Nash network, one of i's link's is to player j', such that $\exists$ a subset $B1 \subset B$ and i's strategy L_i is such that $L_{ij'} > 0$, such that

$$\pi(N/B1 \cup i, L_i, \mathbf{1}) = 0$$

but

$$\pi(B1 \cup i, \mathbf{1} - L_i, \mathbf{1}) \geq 0$$

where again L_j is such that $L_{ji} = 1 - L_{ij}$ and $L_{jk} = 0$ for $k \neq i$.

Now think of the hypothetical scenario of adding another player x to B, then using A3 and A4, he could link to i combining the previous strategies $L_{ij'}$ and $L_{j'i}$ and get a positive profit sponsoring all of the link to i and observing all N. But if he used a combination of strategies L_i^* and L_j^*, he would make negative profits sponsoring all of the link to i and observing all N! (Remember, the two scenarios are exactly alike for x since the profit function does not depend on the rest of the network structure after having accounted for his neighbours.) Which is contradictory and hence it must be that

$$\pi(i, L*_j, \mathbf{1}) \geq 0$$

■

Lemma 3 *If R1 holds for a block, there must be a network which satisfies R1 which is of the form a star.*

Proof. Using the above lemma, we know that for each player has a feasible strategy L^* that allows him to profitably link to all other players using a single link. Or we can allow one player to be the central player and be the counter-party to all of these offers $L*$ and this central player will also make non-negative profits using the assumption of a sum of profitable strategies is a profitable strategy. ■

Lemma 4 *Suppose there is a single block. Starting with any arbitrary network, if R1 holds, this block will be connected.*

Proof. If we start at an empty network, we know from the previous lemma that a player can initiate a star network.

If we start at any other network, first all non-profitable links will be broken off. (Since this is an infinite process this will indeed be the case eventually.) Since there is only one block, the commitments will converge $\theta = 1$.

If there are no profitable links, we return to the empty network. If not, suppose it is the case that players i and j have a profitable link, but k is a singleton. But then k can use a combination of the strategies l_{ij} and l_{ji} to sponsor an entire link to either one of i or j ■

Lemma 5 *If the network is connected, it will converge to a network, where each individual block is internally connected.*

Proof. Suppose the network is connected but $i, i' \in B$ are not internally connected. Suppose wlog, i only has an external link to j and i' only has an external link to j'. If j is the player chosen to act, he will extract the maximum link contribution from i, leaving i with zero profits. The if i is chosen to act, he could increase his profits by switching to $\theta_i' = 1$ and a link to i'. ■

Lemma 6 *The network will transition from separated to connected only if the connected network is such that, breaking the inter-block links and giving everyone a theta of 1 is also a static Nash equilibrium.*

Proof. A network will converge to connected from separated only if the player receiving the offer for the external link can support his internal link under the changed lower θ. In that sense, we need that the minimum this player pays when he is forming only the internal link be still profitable when he forms the external link. The basic problem is that the player offering the link can change his internal link contribution simultaneously, but the player receiving the external link offer cannot. ■

B Proof for Fluid Identity, Choice of Identity has no Direct Impact on Utility, Static Game

The proof of the proposition on Nash equilibrium in the case with fluid identity with identity not directly affecting the utility is presented in the form of the following lemmas.

Lemma 7 *In a Separated Nash equilibrium, players making just one link, should be making equal profits.*

Proof. Suppose $i \in B_1$ participates in just one link with $j \in B_1$. Similarly let there be a link between $i', j' \in B_2$. Now suppose that i' makes higher profits than i. i can then switch identity and make the same offer to j' as i' or $l_{ij'} = l_{i'j'}$ and this strategy will give him higher profits than before. Moreover, if to this strategy, he adds j' strategy, he will make it feasible and since j' link to i' must profitable, in all i will now have a feasible strategy making strictly more than before.

■

Lemma 8 *A Separated Nash equilibrium must have all players making equal profits.*

Proof. Suppose not. Suppose $i \in B_1$ makes higher profits than $i' \in B_2$ who participates in a single link. If i links to $j \in B_1$ and this individual link is profitable (one of i's links must be profitable, so that in all he makes profits), then using the same reasoning as before, i' could profitable deviate to switching identity and forming a link with the other identity block. ■

Lemma 9 *In a Separated Nash equilibrium, the profit from deviation and adding a link to the other block must be the same as the profits made by the players making just one link*

Proof.

We know that for a player making single link in one of the blocks, it must be more profitable to stay in the current block that to change identity and sponsor an entire link to the other block. But since the players making a single link in any block must make the same profits, the lemma follows.

■

Lemma 10 *A Separated Nash equilibrium must have all players making zero profits.*

Proof. Suppose $i \in B_1$ makes single link to i^1. Suppose i makes positive profits, $\pi_i > 0$. In any case, i^1 must make zero profits from this link to i, otherwise the total link between i and i^1 will become more profitable than the profit of i. Which would mean that a player from the other block who makes a single link and the same profits as π_i, would want to deviate to make a link with i. But in all i^1 must make the same profits as i, otherwise he could deviate to join the other block. So i^1 must have a profitable strategy, say with i^2 and i^3. The total profits from these two links must be π_i and both link individually also yield profits. Now i^2 must also make exactly π_i profits; and since from his link with i^1 he makes less than π_i, he must be making at least one more profitable link. Any player with whom he makes a profitable link, will again need to make other profitable links to have total profits of π_i. But this will lead into an infinite cycle given that we know player's who make a single link make π_i from that link. In other words it must be that $\pi_i = 0$. ■

Lemma 11 *Identity Blocks B_1 and B_2 will be supported as Connected Nash network if for some $i, i' \in B_1$ and $j, j' \in B_2$, we have*

$$\pi(N, L_i, \Theta, I) > 0, \pi(N, L_{i'}, \Theta, I) > 0, \pi(N, L_j, \Theta, I) > 0, \pi(N, L_{j'}, \Theta, I) > 0$$

where $\theta_i, \theta_j \leq 1, \theta_{i'} = \theta_{j'} = 1$, $l_{ij}, l_{ji}, l_{ii'}, l_{i'i}, l_{jj'}, l_{j'j} > 0$, $l_{ij} + l_{ji} \geq 1$, $l_{ii'} + l_{i'i} \geq 1$ and $l_{jj'} + l_{j'j} \geq 1$. And none of these players can do better by breaking one of these links or switching identity.

Proof. For a partition to be supported as Connected Nash Network, we need at least one player each from both blocks that is participating in an external as well as internal link. And we must have both these contributions strictly positive. These players must not want to severe either connection or to switch identity. Further more, the players linking to these players making external links should better off keeping those links than severing them. ■

C Proof for Fluid Identity, Choice of Identity has no Direct Impact on Utility, Dynamic Results

Lemma 12 ***Separated Identity Blocks*** *will not survive the dynamic process.*

Proof. Let $i \in B_1$ be player who makes only one link with $i' \in B_1$. i' offers $l_{i'i}$ for this link which must be profitable to him, by A4. If some player $j \in B_2$ offers $l_{ji} = l_{i'i}$ to i, it must be profitable for j, by A3. But if i switches to linking with j and changing his identity, he has the same costs as before, but he is connected to a 'bigger' block now.Or i will switch to a link with j, and the block B_1 will unravel and we will be left with one identity for all players. ■

Lemma 13 *In* ***Connected Identity Blocks****, the player making external links will participate in at most one internal link.*

Proof. Suppose i makes external links and also internal links with say i' and i''. If i is chosen to act at some point, he will extract the maximum possible from both i' and i''. But that leaves i' with the profitable deviation of switching to a link with i'' and offering the same as to i or offering $l_{i'i''} = l_{i'i}$. The deviation is profitable because i' now has the same benefits as before but by making a cheaper link. i'' would choose to accept this offer rather than not be linked to i' at all. ■

Lemma 14 *In* ***Connected Identity Blocks****, there will be exactly one link connecting the two identity blocks.*

Proof. Suppose $i, i' \in B_1$ both participate in external links. Then it must be that they are connected via these external links. Then either one of them has the profitable deviation of switching to a link with the other and breaking the external link. ■

Lemma 15 ***Connected Identity Blocks*** *will not survive the dynamic process.*

Proof. Consider the case where $i \in B_1$ participates in one external link with $j \in B_2$ and one internal link with $i' \in B_1$. Since i does not want to switch identity while keeping the same link strategy, it must be that $l_{ii'} > l_{ij}$. let's assume wlog that $l_{ij} \geq 1/2$. But if i were to switch his identity, there would be one more person in the block B_2 and so i' would be willing to change his offer to $l'_{i'i}$ such that $l'_{i'i} > l_{ij} \geq 1/2$ to maintain the link with i and hence with the other identity block. Or in other words, i could profitably move to the other identity and pay less than 1/2 for his now external link with i' and more than 1/2 for his now internal link with j. Earlier he was paying more than 1/2 for both links. But then i' will have a similar incentive to switch identity and the process will converge to a unique identity for all. ■

D Proof for Fluid Identity, Choice of Identity has Direct Impact on Utility, Static Game

Again the proof for the proposition on Nash networks is presented in the following lemmas.

Lemma 16 *Assuming A1-A5, for all $b_i \leq n/2$, $\overline{\psi(b_i)} \geq \underline{\psi(b_i)}$*

Proof. Wlog assume that player j makes $\pi^e(n-b_i) = \pi(n-b_i, L_j, \Theta, I)$, where L_j involves a link offer to only one player k to access the entire block. Also, player k's profit from this link to j must be positive, or $\pi_{kj}(1, L_{kj}, \Theta, I) \geq 0$. Now, playing L_j and L_{kj} will yield higher profits than $\pi^e(n-b_i)$ by A3, but this addition will mean that a player observing b_i+1 players and paying for the cost of one entire link is better off than making $\pi^e(n-b_i)$. In other words we have:

$$\pi^s(n-b_i+1) \geq \pi^e(n-b_i)$$

Similarly, we can show

$$\pi^e(b_i) \leq \pi^s(b_i+1)$$

Together they imply $\overline{\psi(b_i)} \geq \underline{\psi(b_i)}$ ■

Lemma 17 *If a Separated Nash equilibrium exists at $b_1, n-b_1$*

$$\pi^s(b_1+1) > \pi^e(b_1) > \pi^s(b_1)$$

Proof. The first inequality follows from the previous lemma. The second inequality must hold from the definition of $\pi^e(.)$ ■

Lemma 18 *If there exists a Separated Nash equilibrium, it cannot be that $\phi(.)$ is always increasing.*

Proof. From the previous lemma, we know that the network profits from sponsoring an entire link to any larger block size are greater than being the lowest profit maker in a smaller block. If to that, we add higher benefits to identity to larger blocks, the lowest profit maker in the smaller block will always want to deviate to sponsoring an entire link to the larger block if $\phi(.)$ were always increasing. ■

Lemma 19 *The only Nash equilibrium if ϕ is symmetric around $n/2$ will be the connected equilibrium with a single identity chosen.*

Proof.

Consider any partition $b_1 \leq n/2$ and $n - b_1 \geq n/2$. Since ϕ is symmetric, it must be that ψ is always at zero. Or in other words, its always profitable for the player of the smaller block to deviate to the larger block, since, $\overline{\psi} > 0$ by the first lemma. Since this is true for any arbitrary division, it must mean that a Separated Nash equilibrium with two identities is not possible. ■

Lemma 20 *For a ϕ that is concave, at most one partition could be supported as Separated Nash equilibrium. Further, for a partition to exist, the peak must be before $n/2$. The smaller block size will be bigger than the block size at which the peak occurs. (this includes the case of a continuously decreasing phi, here the peak would be at $b_i = 1$)*

Proof.

Suppose the peak in ϕ occurs at x_1. Then we will see ψ will also increase till $min(x_1, n - x_1 + 1)$ and then decrease. Moreover, if $x_1 \leq n/2$, then ψ will be positive throughout; but if $x_1 \geq n/2$, then ψ will be negative throughout and there will be no Separated Nash equilibrium possible. If $x_1 \leq n/2$, then a necessary condition for the equilibrium is that $\psi(x_1) \geq \overline{\psi(x_1)}$ and either $\psi(n/2) \geq \underline{\psi(n/2)}$ or $\psi(n/2) \geq \overline{\psi(n/2)}$. ■

Lemma 21 *For a ϕ that is convex, at most one partition could be supported as Separated Nash equilibrium. Further, for a partition to exist, the trough must be after $n/2$.*

Proof.

Suppose the trough in ϕ occurs at x_1, then we will see ψ will also increase till $min(x_1, n - x_1 + 1)$ and then decrease. Moreover, if $x_1 \leq n/2$, then ψ will be negative throughout and there will be Separated Nash equilibrium possible; but if $x_1 \geq n/2$, then ψ will be positive throughout. ■

Lemma 22 *If ϕ is such that it can be partitioned into regions that are concave around peaks and convex around troughs, the number of Separated Nash equilibrium will be at most the number of peaks in ϕ.*

Proof.

Since for a Separated Nash equilibrium to exist, ψ has to cross from being above $\overline{\psi}$ to being below $\underline{\psi}$, we must have that the number of Nash equilibrium is less than the number of peaks in ψ. Now suppose that ϕ has four peaks, moreover it has peaks at $x_1, x_2 \leq n/2$ and troughs at $x_3, x_4 \geq n/2$. It follows that ψ will have peaks at most at $\{x_1, x_2, n - x_3 + 1, n - x_4 + 1\}$

Moreover, if the the function ϕ has troughs at $y_1, y_2 \leq n/2$ and peaks at $y_3, y_4 \geq n/2$, then ψ will have troughs at most at $\{y_1, y_2, n - y_3 + 1, n - y_4 + 1\}$

The necessary condition for a Separated Nash equilibrium to exist between a peak of ψ, say x_1, and its corresponding trough. say y_2, is that $\psi(x_1) \geq \overline{\psi(x_1)}$ and $\psi(y_1) \leq \underline{\psi(y_1)}$. ■

E Proof for Fluid Identity, Choice of Identity has Direct Impact on Utility, Dynamic Result

Lemma 23 *Dynamic equilibria are also Nash equilibria.*

Proof. We will show that Separated Dynamic equilibria are also Separated Nash equilibria. The proof for the connected case is very similar and is omitted.

We must have

$$\overline{\psi^{DE}(b_i)} \geq \overline{\psi(b_i)}$$

because

$$\overline{\psi^{DE}(b_i)} - \overline{\psi(b_i)} = \pi(n - b_i + 1, L^{min}, \Theta, I^i) - \pi^s(n - b_i + 1) + \pi^e(b_i) - \pi(b_i, L^{min}, \Theta, I)$$

or

$$\overline{\psi^{DE}(b_i)} - \overline{\psi(b_i)} = \pi(b_i, L^{min}, \Theta, I^i) - \pi(b_i, L^s, \Theta, I^i) + \pi^e(b_i) - \pi(b_i, L^{min}, \Theta, I)$$

or

$$\overline{\psi^{DE}(b_i)} - \overline{\psi(b_i)} = \pi^e(b_i) - \pi(b_i, L^s, \Theta, I^i) > 0$$

Similarly, we must have

$$\underline{\psi^{DE}(b_i)} \leq \underline{\psi(b_i)}$$

■

Lemma 24 *Any subgroup of an identity block can deviate to switching identity and linking to the other identity block.*

Proof. Consider a network where b_1 players have chosen identity characteristic c_1 and b_2 players have chosen identity characteristic c_2. Let us consider a deviation by a subgroup b_1' of b_1 to identity characteristic c_2.

Suppose the block b_1 is arranged in such a way that player $i \in B_1$ is the center of a star with b_1' spikes and another link to the rest of the players. If i is the chosen player in any period, then he can retain links with b_1' players on the condition that they switch identity to c_2, sever his link with the rest of his block and offer a new link to the other identity block.

If b_1 does not already have that form, then we know with positive probability it will attain that form. Consider any other player $j \in B_1$ who wlog forms a single link with j'. In any period if j is chosen to act, one of his best responses will be to sever link with j' and offer to link with i. Since j forms a single link, linking to any player in his identity block is a best response. By A3, this link offer will be acceptable by i. And continuing thus, i would in some periods end up as the center of a star with b_1' spokes which would then have to choice to deviate to a link with the other identity block. ■

Lemma 25 *The dynamic game will not converge to block sizes $b_1 \leq b_2$, if there exists some $n_3 > b_2$ such that $\phi(n_3) \geq \phi(b_1)$*

Proof. If it were so, then $n_3 - b_2$ players from B_1 would migrate to the bigger block to make higher network profits *and* higher profits from identity.

■

Lemma 26 *If*

$$\pi(B_1, L_1, \Theta, I) - \pi(B_1, L_2, \Theta, I) = \pi(B_2, L_1, \Theta, I) - \pi(B_2, L_2, \Theta, I)$$

then, the dynamic game will converge to separated blocks of $b_1 \leq b_2$ if for any $b_1' \geq b_1$ and/or any $b_2' \geq b_2$, it were true that

$$\pi(b_1', 0, \Theta, I^{b_1', n-b_1'}) + \phi(b_1') \leq \pi(b_2, 0, \Theta, I) + \phi(b_2)$$

$$\pi(b_2', 0, \Theta, I^{n-b_2', b_2'}) + \phi(b_2') \leq \pi(b_1, 0, \Theta, I) + \phi(b_1)$$

($I^{x,y}$ denotes the identity profile where x players have identity c_1 and y players have identity c_2)

Proof. From the previous lemmas we know that a move to any partition that involves players migrating from one of the current blocks to the other is not possible. The assumption ensure this to be true for all strategy profiles, in particular for the profile at which no costs are bourne. ■

Lemma 27 *A necessary condition for the dynamic game to converge to connected identity blocks of sizes $b_1 \leq b_2$ if for any $b_1' \geq b_1$ and/or any $b_2' \geq b_2$, it were true that*

$$\phi(b_1') \leq \phi(b_2)$$

$$\phi(b_2') \leq \phi(b_1)$$

($I^{x,y}$ denotes the identity profile where x players have identity c_1 and y players have identity c_2)

Proof. These conditions must hold because any player/block choosing to deviate to the other identity block will make the same network profits, they will be making the decision based only on the identity benefits.
■

Lemma 28 *If a dynamic equilibrium exists, it is unique.*

Proof. Suppose a Separated Dynamic equilibrium exists at b_1, b_2 as well at n_1, n_2. We know that either $b_2 < n_1 < n_2 < b_2$ or $n_1 < b_1 < b_2 < n_2$. Suppose the former case is true , then we show that n_1, n_2 cannot be a Separated Dynamic equilibrium. This is so because for b_1, b_2 to be a Separated Dynamic equilibrium, it must be that the profits at n_1 are less than profits at b_2. In other words, some players from n_1 will wish to join the other identity block to form a block of size b_2. Similarly, in the latter case b_1, b_2 cannot be an equilibrium.

Suppose a Connected Dynamic equilibrium exists, then players will keep moving between identities till no improvements can be made on identity benefits. ■

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